

# Transient Analysis Of A Spatial Power Combining Amplifier

C. E. Christoffersen, S. Nakazawa, M. A. Summers and M. B. Steer<sup>†</sup>  
 Department of Electrical and Computer Engineering, North Carolina State  
 University.

<sup>†</sup> Institute of Microwaves and Photonics, School of Electronic and Electrical  
 Engineering, University of Leeds.

*Abstract*— An integrated electromagnetic and circuit simulation environment is developed for the transient simulation of spatial power combining systems. The analysis incorporates surface modes, nonuniform excitation and full nonlinear effects. A state-variable approach makes the transient analysis very robust. The simulation tools are used to predict the performance of a 2 by 2 quasi-optical grid amplifier.

## I. INTRODUCTION

Quasi-optical systems combine the power of numerous solid state devices in free space. A typical quasi-optical system is the grid system shown in Fig. 1 where a large number of active device are distributed on the grid surface. The grid is excited by a horn and lens system which concentrates the incident field on the grid and polarizers are used to isolate the input and output.

Electromagnetic models are required to design these spatially distributed systems. The electromagnetic simulator produces the multi-port admittance matrix of the passive grid structure for inclusion in a microwave circuit simulation program. The admittance matrix produced is port based due to the lack of a global reference node in the spatially distributed structure. The multiport admittance matrix, along with a nonlinear active device

model are used here in a transient circuit simulator to model the performance of the spatial power combining system.

Transient analysis of distributed microwave circuits is complicated by the inability of frequency independent primitives (such as resistors, inductors and capacitors) to model distributed circuits. More generally, the linear part of a microwave circuit is described in the frequency domain by network parameters especially where numerical field analysis is used to model a spatially distributed structure. Inverse Fourier transformation of these network parameters yields the impulse response of the linear circuit. This has been used with convolution to achieve transient analysis of distributed circuits [2], [3].

This paper develops a convolution-based transient analysis which uses state variables instead of node voltages to capture the nonlinear response. This has two desirable effects. First, the number of state variables required is usually less than the number of nodes of the nonlinear elements and so fewer quantities need to be recorded and convolved. Second, parameterized device models can be used as the nonlinearity is not restricted to the form of current as a nonlinear function of voltage. Parameterized device models result in much better convergence properties of the nonlinear system of equations solved at each time step. Thus, there is no need for analytical derivatives, simplifying model development. In a more general context this paper is a development in the area of device-circuit-field

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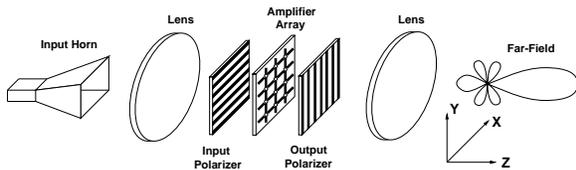


Fig. 1. Quasi-optical lens system configuration with a grid amplifier array and polarizers.

interaction with the transformation of an electromagnetic simulation of a distributed structure into a circuit element. The electromagnetic model is reduced to terminal characteristics at defined ports and represented as a port-based admittance matrix. It is important to note this matrix is port-based since there is not a global reference node [9]. Therefore, information about local reference terminals and groups is also passed to the circuit simulator [10].

## II. FORMULATION OF THE TRANSIENT ANALYSIS

As it is now conventional for distributed microwave circuits, the circuit is partitioned into linear and nonlinear subcircuits [4]. The core of the method presented is that state variables are used to describe the nonlinear dependence in the nonlinear subcircuit. This enables more flexibility in writing the nonlinear element models so that better convergence might be achieved [5].

The linear subcircuit is characterized in the frequency domain and the other part, which includes the nonlinear devices, is treated in the time domain. From [6], the vector of voltages in the frequency domain is,

$$\mathbf{V}_L(\mathbf{X}, f) = \mathbf{S}_{SV}(f) + \mathbf{M}_{SV}(f) \mathbf{I}_{NL}(\mathbf{X}, f) \quad (1)$$

$\mathbf{M}_{SV}(f)$  is obtained from a modified nodal admittance matrix that supports the concept of local reference nodes [9], [10]. Since the independent sources are more easily handled in the time domain for this kind of analysis, the source vector  $\mathbf{S}_{sv}(f)$  can be assumed to be zero. Expanding the matrix multiplication, each element of the voltage vector

$\mathbf{V}_L(\mathbf{X}, f)$  can be written as:

$$V_{Li} = \sum_{j=1}^{n_S} M_{i,j}(f) I_{NL,j} \quad (2)$$

Rewriting one term of this equation in the time domain and replacing the multiplications with the convolution operation leads to

$$v_{Li,j}(t) = m_{i,j}(t) * i_{NL,j}(t) \quad (3)$$

Now, expanding the convolution operation we get:

$$v_{Li,j}(t) = \int_{-\infty}^t m_{i,j}(\tau) i_{NL,j}(t - \tau) d\tau \quad (4)$$

where the system is assumed to be causal,  $i_{NL,j}(t) = 0$  for  $t \leq 0$ . Numerical evaluation requires discretization as follows. First, each element of the I-FFT of  $\mathbf{M}_{SV}$  has a finite number of components, denoted  $N_T$ . Using the trapezoidal integration rule [8] and using  $m_{i,j}$  obtained from the I-FFT, we obtain Eq. (5). For  $n_t < N_T$  the last term is zero since  $i_j(0) = 0$ . For  $n_t \geq N_T$  the last term is also zero since  $m_{i,j}(n_t) = 0$ .

$$v_{Li,j}(\hat{\mathbf{X}}, n_t) = \begin{cases} \frac{m_{i,j}(0) i_{NL}(\hat{\mathbf{X}}, n_t)}{2} \\ + \sum_{n_\tau=1}^{n_t-1} m_{i,j}(n_\tau) i_j(n_t - n_\tau), \\ \text{if } n_t < N_T \\ \frac{m_{i,j}(0) i_{NL}(\hat{\mathbf{X}}, n_t)}{2} \\ + \sum_{n_\tau=1}^{N_T} m_{i,j}(n_\tau) i_j(n_t - n_\tau), \\ \text{if } n_t \geq N_T \end{cases} \quad (5)$$

Note that in all but one of the convolution sum terms,  $i_{NL} = i_L = i$ , so most of the convolution sum can be performed before beginning the iterations to solve the nonlinear system of equations. The following error function is solved at each time step

$$F_i(\hat{\mathbf{X}}) = V_{Li}(\hat{\mathbf{X}}) - V_{NLi}(\hat{\mathbf{X}}) \quad (6)$$

### Impulse Response Determination

The impulse response is obtained as the inverse Fourier transformation of each element of  $\mathbf{M}_{SV}$ . The transform requires special characteristics of

the frequency domain variables at high frequencies so that aliasing is minimized. This can be corrected by cascading a resistive augmentation circuit with the linear circuit and so ensure finite parameters [1], [2]. Being resistive, the effect of the augmentation network can be removed in transient simulation as the resistors are unaffected by the Fourier transformation.

### III. SIMULATION RESULTS

The simulation tools were used to model the non-linear performance of a 2 by 2 quasi-optical grid amplifier system described in [7]. The layout is shown in Fig. 2.

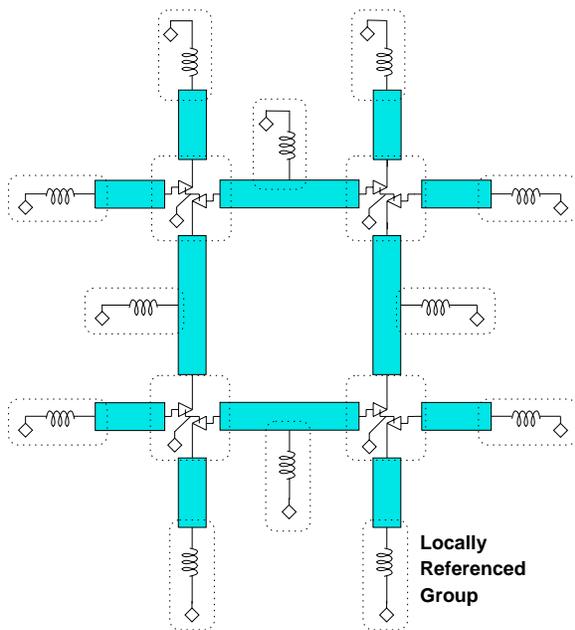


Fig. 2. Layout for 2x2 quasi-optical grid amplifier.

A horn-to-horn simulation of the grid amplifier system described above was performed. The grid structure was modeled using a MOM field simulator [11] to generate the multi-port admittance matrix and excitation currents for the grid structure. The transient circuit simulation was used to calculate the voltage waveforms at the ports of the grid. The horns are included in the simulation by modeling the horns using their far-field gain. The gain

of the horns is used to calculate the field incident on the grid and the power received from the field radiated by the grid amplifier.

The transient simulation of the turn-on transient of the grid amplifier without excitation is shown in Fig. 3. The system is stiff, *i.e.*, it has wide-varying time constants. Therefore, the time step must be kept small, but the time to reach the DC operating point is long. The simulation time was 64 hours on a Sun UltraSPARC 1 workstation, clocked at 143 MHz. Fig. 4 presents another simulation using a smaller time step for the the first 4 ns with excitation applied after 2 ns. The excitation frequency is 5.4 GHz. Simulation time was 2 hours. Fig. 5 shows the steady state portion of the transient waveform with excitation applied compared to a harmonic balance simulation previously reported and compared to measurements by us [7]. In this case, the circuit was already at the operating point at the starting of the transient simulation. The simulation time was 10 hours.

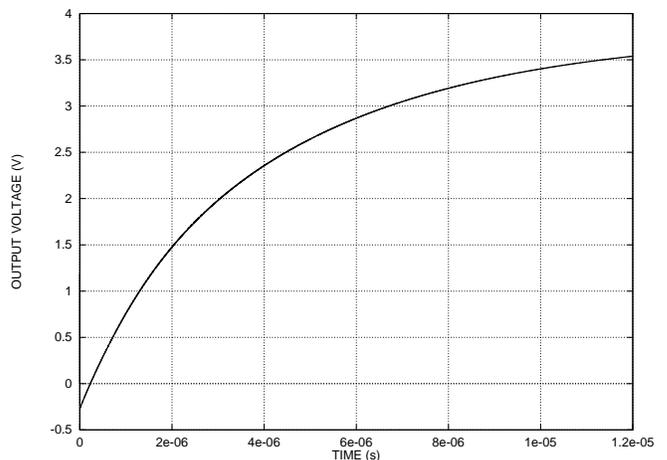


Fig. 3. Transient response at one of the MMIC output ports using a large time step.

### IV. CONCLUSIONS

The importance of the work presented here was the development of a state-variable formulation for transient analysis of distributed circuits using the impulse response of the linear subnetwork and con-

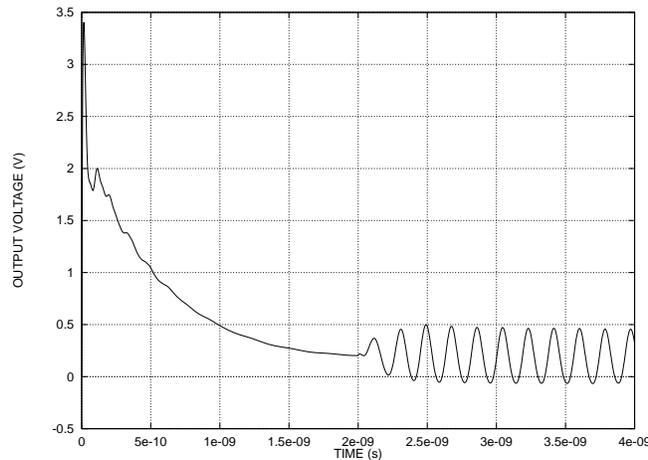


Fig. 4. Zoom of the transient response at one of the MMIC output ports using a small time step.

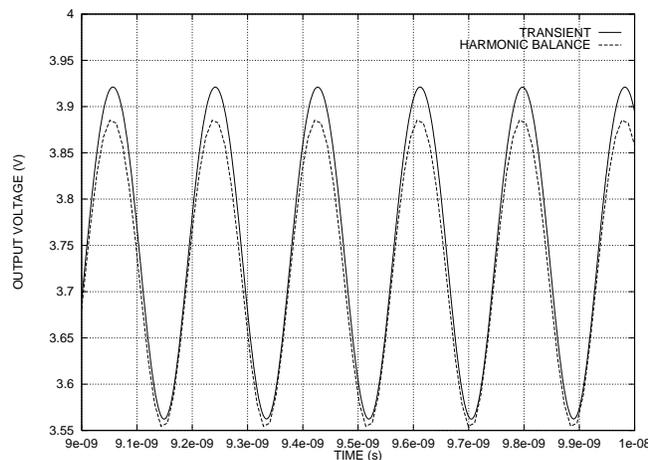


Fig. 5. Comparison between transient and harmonic balance output waveforms.

olution. Thus transient simulation technique developed here is targeted for stability investigations and analysis of circuits with tightly coupled circuit and field interactions.

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