WAVE-BASED TRANSIENT ANALYSIS USING BLOCK NEWTON-JACOBI

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ABSTRACT

A recently introduced wave-based transient analysis uses relaxation and thus does not require large matrix decompositions at each nonlinear iteration. The use of waves results in guaranteed convergence for any linear passive circuit and some types of nonlinear circuits, but the convergence rate can not be controlled. In this work, the wave-based transient analysis is re-formulated using a block Newton-Jacobi approach and the convergence properties of the original and new formulations are compared with the simulation of two microwave circuits.

Index Terms— transient analysis, microwave circuits, wave digital filters, state variables, Newton-Jacobi, scattering matrix.

1. INTRODUCTION

Recently a new wave-based transient analysis aimed to microwave circuits was introduced in Reference [1]. This transient analysis formulation is based on fixed-point iterations of waves at the ports of nonlinear devices. Unlike Newton's method, fixed-point (or relaxation) methods do not require a matrix decomposition of a large Jacobian matrix at each nonlinear iteration. Also fixed-point methods exploit the latency of the circuit by decoupling it into smaller pieces and solving each piece independently. Thus they are more suitable to be implemented in parallel computer systems. Different relaxation methods have been previously proposed for circuit simulation [5]. The greatest advantage of the approach presented in [1] compared to the traditional fixed-point iteration approaches using voltages and currents is that the power delivered to nonlinear devices at any iteration is bounded [2]. Convergence is guaranteed for any linear passive circuit and some types of nonlinear circuits. However the convergence rate is in general slower than with Newton method.

Transient simulation of circuits using waves have been proposed in a few works (Ref. [4] is one example) in the context of wave digital filters (WDF) [3]. WDF are discrete structures that mimic an analog reference circuit, initially employed to implement digital filters but they can be applied to model any circuit. Circuit simulation using relaxation in the WDF context was explored in References [4] and [2]. The approach in [2] allows nonlinear devices to be modelled using voltages and currents, and thus it is more suitable for implementation in a circuit simulator. In Ref. [1], the method proposed in [2] was further developed with parameterized nonlinear device models, implemented in the *f*REEDA™ [6] simulator and tested with a wider variety of circuits.

This paper proposes for the first time to use a block Newton-Jacobi approach to accelerate the convergence of the nonlinear equations that arise in the transient analysis aimed to microwave circuits presented in [1]. Other methods to solve these equations exist but the Newton-Jacobi approach is attractive because its implementation requires only minor modifications to the original fixed-point approach and shares the fixed-point-method advantage of exploiting the latency of the circuit. The convergence properties of the new and the original approach are compared in this work. Relevant equations and the iteration scheme for Wave-Transient analysis is presented in Section 2. Section 3 describes the proposed Newton-Jacobi approach. It is shown in this section that the proposed approach requires few additional computations compared to the original fixed-point approach. Simulation results for two microwave circuits are presented and discussed in Section 4.

2. FORMULATION OF WAVE-BASED TRANSIENT ANALYSIS

Equations are formulated following the state-variable approach [7] used in *f*REEDA™. The circuit is partitioned in sources, linear and nonlinear parts (Fig. 1). For each nonlinear element ports are defined with one terminal taken as the reference. The linear network is assumed to be passive.

The nonlinear subnetwork is described by the following

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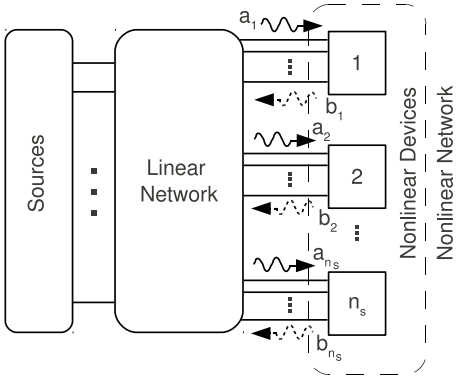

Fig. 1. Network partition

parametric equations [7]:

$$\mathbf{v}_{NL}(t) = \mathbf{v}\left(\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \ldots, \frac{d^m\mathbf{x}}{dt^m}, \mathbf{x}_D(t)\right) \quad (1)$$

$$\mathbf{i}_{NL}(t) = \mathbf{i}\left(\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \ldots, \frac{d^m\mathbf{x}}{dt^m}, \mathbf{x}_D(t)\right) \quad (2)$$

where $\mathbf{v}_{NL}(t)$, $\mathbf{i}_{NL}(t)$ are vectors of voltages and currents at the ports of the nonlinear network, $\mathbf{x}(t)$ is a vector of state variables and $\mathbf{x}_D(t)$ is a vector of time-delayed state variables. All vectors in Eqs. (1) and (2) have the same size equal to the number of ports of the nonlinear network (n_s).

Applying numerical integration and discrete convolution on the standard MNA equation and employing connectivity information between the linear and nonlinear subnetwork the following error function is obtained (the full derivation can be found in [1])

$$\mathbf{s}_{sv,n} - \mathbf{M}_{sv}\mathbf{i}_{NL}(\mathbf{x}_n) - \mathbf{v}_{NL}(\mathbf{x}_n) = 0 \ , \quad (3)$$

where $\mathbf{s}_{sv,n}$ is a vector that accounts for the sources and the previous history of the network. $\mathbf{s}_{sv,n}$ depends only on quantities known at the nth time step. The $\mathbf{M}_{sv}$ matrix is constant and represents the linear network. The size of the algebraic system of nonlinear equations (3) is $n_s \times n_s$.

For each nonlinear port, an arbitrary reference resistance, R_j with j the port number, is chosen. The incident and reflected power waves at Port j (a_j and b_j, respectively) are defined as follows [3],

$$a_j = \frac{v_j + R_j i_j}{2\sqrt{R_j}} \ , \ b_j = \frac{v_j - R_j i_j}{2\sqrt{R_j}} \ ,$$

where v_j and i_j are the instantaneous values of the voltage and current at the port. The total voltage (v_j) and current (i_j) at Port j can be expressed as

$$v_j = \sqrt{R_j}(a_j + b_j) \ , \ \ i_j = \frac{(a_j - b_j)}{\sqrt{R_j}} \ .$$

Thus voltage and current vectors corresponding to all nonlinear device ports are related to the power wave vectors ($\mathbf{a}$ and $\mathbf{b}$) as follows:

$$\mathbf{v}_{NL} = \mathbf{D}(\mathbf{a} + \mathbf{b}) \ , \quad (4)$$

$$\mathbf{i}_{NL} = \mathbf{D}^{-1}(\mathbf{a} - \mathbf{b}) \ , \quad (5)$$

where $\mathbf{D}$ is a diagonal matrix with the square roots of corresponding reference port resistances in the main diagonal.

The relaxation method is based on propagating reflections of waves between the linear and nonlinear subnetworks. Assume an initial vector of reflected waves ($\mathbf{b}_n^{(k)}$) is known, where (k) denotes the iteration number. The corresponding waves sent by the linear network ($\mathbf{a}_n^{(k+1)}$) can be calculated by replacing Eqs. (4) and (5) in Eq. (3),

$$\mathbf{a}_n^{(k+1)} = \mathbf{S}\mathbf{b}_n^{(k)} + \mathbf{a}_{0,n} \ , \quad (6)$$

with

$$\mathbf{S} = [\mathbf{M}_{sv}\mathbf{D}^{-1} + \mathbf{D}]^{-1}[\mathbf{M}_{sv}\mathbf{D}^{-1} - \mathbf{D}] \ , \quad (7)$$

$$\mathbf{a}_{0,n} = -[\mathbf{M}_{sv}\mathbf{D}^{-1} + \mathbf{D}]^{-1}\mathbf{s}_{sv,n} \ , \quad (8)$$

here, $\mathbf{S}$ is the scattering matrix of the linear network and $\mathbf{a}_{0,n}$ is the contribution of sources and previous history to the incident waves.

3. BLOCK NEWTON-JACOBI METHOD

The proposed block Newton-Jacobi method is based on splitting the scattering matrix, $\mathbf{S}$ into a sum of a block-diagonal matrix ($\mathbf{S}_B$) and another matrix with zeros in the diagonal blocks ($\mathbf{S}_O$) [2]:

$$\mathbf{S} = \mathbf{S}_B + \mathbf{S}_O \ . \quad (9)$$

Each block in $\mathbf{S}_B$ is associated to one nonlinear device, *i.e.* the rows and columns of each block correspond to the ports of one nonlinear device. The $\mathbf{S}_O$ matrix represents the couplings between nonlinear devices. The basic idea of the approach is to include the corresponding block of $\mathbf{S}_B$ in the equations for each nonlinear device in the following way:

$$\mathbf{v}_{NL}(\mathbf{x}_n) = \mathbf{D}(\mathbf{a}_n - \mathbf{S}_B\mathbf{b}_n + \mathbf{b}_n) \ , \quad (10)$$

$$\mathbf{i}_{NL}(\mathbf{x}_n) = \mathbf{D}^{-1}(\mathbf{a}_n - \mathbf{S}_B\mathbf{b}_n - \mathbf{b}_n) \ . \quad (11)$$

The contribution of $\mathbf{S}_O$ is considered in the main relaxation loop. Thus Eq. (6) is modified as follows:

$$\mathbf{a}_n^{(k+1)} = \mathbf{S}_O\mathbf{b}_n^{(k)} + \mathbf{a}_{0,n} \quad (12)$$

where (k) denotes the relaxation iteration number. Note that if there is no coupling between nonlinear devices (*i.e.*, $\mathbf{S}_O$ is zero) then Eqs. (10) and (11) produce the exact solution of the system. If the coupling is not very strong it is expected

that the solution produced by these equations is a better approximation than the solution obtained by plain relaxation.

Newton's method is used to solve Eqs. (10) and (11). By rearranging them, the error functions, $\mathbf{\Phi}_v(\ )$ and $\mathbf{\Phi}_i(\ )$ are obtained:

$$\mathbf{\Phi}_v(\mathbf{x}_n, \mathbf{b}_n) = \mathbf{D}^{-1}\mathbf{v}_{NL}(\mathbf{x}_n) - \mathbf{a}_n - \mathbf{b}_n(\mathbf{I} - \mathbf{S}_B) \quad (13)$$

$$\mathbf{\Phi}_i(\mathbf{x}_n, \mathbf{b}_n) = \mathbf{D}\mathbf{i}_{NL}(\mathbf{x}_n) - \mathbf{a}_n + \mathbf{b}_n(\mathbf{I} + \mathbf{S}_B) \quad (14)$$

where, $\mathbf{I}$ is the identity matrix. The number of equations and unknowns is $2n_s$. The unknowns are vectors $\mathbf{b}_n$ and $\mathbf{x}_n$. The $\mathbf{a}_n$ vector contains the waves incident to the nonlinear devices and is calculated from Eq. (12). Note that this system of equations is decoupled for each nonlinear device, thus the Jacobian matrices arising from it are block-diagonal with small diagonal blocks, typically no more than 6×6 elements. The Jacobian matrix is as follows:

$$\mathbf{J} = \begin{bmatrix} \mathbf{D}^{-1}\mathbf{J}_V & -(\mathbf{I} - \mathbf{S}_B) \\ \mathbf{D}\mathbf{J}_I & +(\mathbf{I} + \mathbf{S}_B) \end{bmatrix}, \quad (15)$$

where, $\mathbf{J}_V = \partial \mathbf{v}_{NL}/\partial \mathbf{x}_n$ and $\mathbf{J}_I = \partial \mathbf{i}_{NL}/\partial \mathbf{x}_n$. These are the block-diagonal Jacobian matrices that are explicitly available in *f*REEDA™.

The updates from each Newton iteration are calculated as follows:

$$\Delta\mathbf{x}^{(\nu+1)} = \left[(\mathbf{I} - \mathbf{S}_B)\,\mathbf{D}^{-1}\mathbf{J}_V + (\mathbf{I} + \mathbf{S}_B)\,\mathbf{D}\mathbf{J}_I\right]^{-1} \left\{-\left[\mathbf{I} + \mathbf{S}_B\right]\mathbf{\Phi}_i^{(\nu)} - \left[\mathbf{I} - \mathbf{S}_B\right]\mathbf{\Phi}_v^{(\nu)}\right\}, \quad (16)$$

$$\Delta\mathbf{b}^{(\nu+1)} = \left[\mathbf{I} - \mathbf{S}_B\right]^{-1}\left(-\mathbf{\Phi}_i^{(\nu)} - \mathbf{D}\mathbf{J}_I\Delta\mathbf{x}^{(\nu+1)}\right), \quad (17)$$

were (ν) is the Newton iteration number, $\Delta\mathbf{x}$ and $\Delta\mathbf{b}$ are the updates of state variables and reflected waves at each Newton iteration. Note that Newton method is used only to approximate the solution for one iteration. Most of the time only a few iterations (often only one) are required to reach an acceptable tolerance, as will be shown in Section 4.

One additional matrix decomposition ($[\mathbf{I} - \mathbf{S}_B]^{-1}$) that was not required with plain relaxation is required for calculating $\Delta\mathbf{b}$ with the Newton-Jacobi approach. But this matrix is small and the decomposition is needed only once for the whole simulation, as long as the time step is kept constant and therefore introduces very little overhead.

The effect of solving Eqs. (10) and (11) using Newton method can be summarized as a nonlinear vector function $\mathbf{G}(\ )$,

$$\mathbf{b}_n = \mathbf{G}(\mathbf{a}_n)\ . \quad (18)$$

The $\mathbf{x}$ vector is not included here since $\mathbf{x}$ is only required at the nonlinear device level (Eqs. (16) and (17)) and is not required in the main iteration loop. Thus the Newton-Jacobi scheme can be expressed as a fixed-point scheme by combining Eqs. (12) and (18):

$$\mathbf{b}_n^{(k+1)} \quad = \quad \mathbf{G}(\mathbf{S}_O\mathbf{b}_n^{(k)} + \mathbf{a}_{0,n})\ . \quad (19)$$

In addition a vector extrapolation method called Minimum Polynomial Extrapolation (MPE) [8] is applied to the vector sequence generated by Eq. (19) to accelerate the convergence rate. The theory for MPE is covered in Ref. [1]. The algorithm for the analysis is provided in Alg. 1. Here, K is the number of vectors to extrapolate, tol is the predefined tolerance, h is the time step size and t_{end} is the simulation end time. All those are set as simulation parameters. tol_{nl} is the predefined tolerance for nonlinear iteration ($tol_{nl} >> tol$).

Algorithm 1 Newton-Jacobi Wave-Based Transient Analysis

```
calculate M_sv,
calculate S using Eq. (7),
repeat
    initial guess is set to the results in previous step,
    update s_{sv,n},
    calculate a_{0,n} using Eq. (8),
    repeat
        calculate a using Eq. (12).
        repeat
            apply Eq. (16) and Eq. (17) to obtain the approximation of b.
        until (error_nl ≤ tol_nl)
        update b for K iterations,
        then extrapolate.
    until (error ≤ tol)
    increase time, t = t + h.
until (t < t_end)
```

4. SIMULATION RESULTS

The proposed block Newton-Jacobi wave-based transient analysis was implemented in the *f*REEDA™ simulator in two analysis types: *WaveTran2* and *WaveTran2-E*. *WaveTran2* uses **LU** decomposition in Eq. (17) at each iteration. *WaveTran2-E* is more efficiently implemented and uses a matrix-vector multiplication algorithm that considers the particular structure of $\mathbf{S}_O$ in Eq. (12) and Cholesky decomposition in Eq. (17) that is performed just once. The performance of the proposed approach is compared with the original (plain relaxation) wave-based transient analysis, type: *WaveTran* [1], and the state-variable transient analysis, type: *Tran2* which uses the formulation of Eq. (3) [7]. Results from Tran2 are assumed to be correct, as these analysis has been previously verified against measurements and other circuit simulators [9,10]. The performance of the proposed method for different circuits is summarized in Table 1. All simulations use fixed time step and $tol = 10^{-8}$ and $tol_{nl} = 0.5$.

Circuit 1, shown in Fig. 2, is a nonlinear transmission line, or *soliton* line [10], composed of 47 diodes and 48 lossy transmission lines. Circuit 2 is shown in Fig. 3 and is composed of 5 MMIC LNA [9] each connected to a soliton line (Circuit 1) at the output stage, each LNA fed with a different input frequency. The size of the modified nodal admittance matrix (MNAM) in each circuit are given in the table to give an idea of the problem size. Values in the table are average per time step except Newton iteration, this is the average value per

Table 1. Summary of simulation results

Circuit	Soliton (Fig. 2)	Multi-MMIC (Fig. 3)
NL ports (n_s)	47	252
MNAM Size	2017	12860
Ref. Port Res. (Ω)	440	50
In. power level (dBm)	30	19
Time Steps	5000	4000
Iter. (WaveTran)	5	13
Iter. (WaveTran2)	4	5
Newton Iter. (WaveTran2)	1.13	1.00
WaveTran Sim. T. (s)	51.7	734.93
WaveTran2 Sim. T. (s)	57.54	**458.33**
WaveTran2-E Sim. T. (s)	**44**	**355.42**
Tran2 Sim. T. (s)	24.03	1576

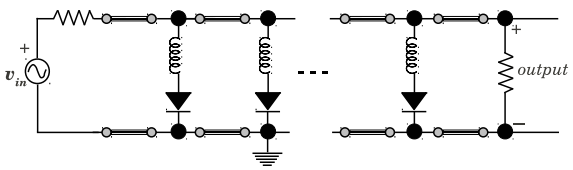

Fig. 2. Soliton-line schematic

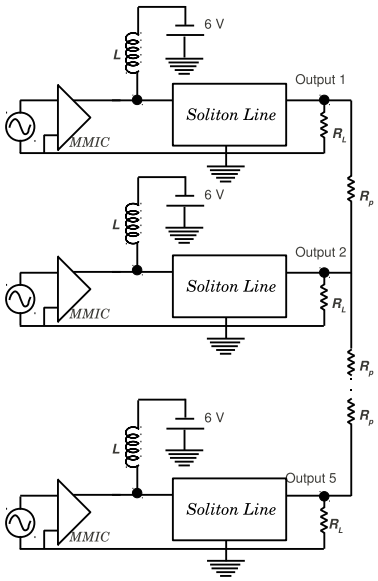

Fig. 3. Multi MMIC-Soliton-line schematic

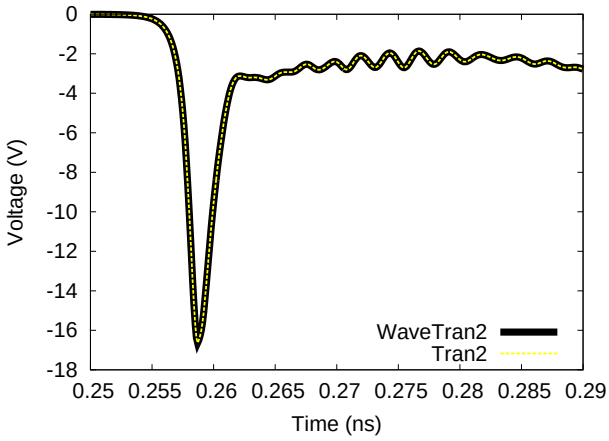

Fig. 4. Output voltage of Soliton Line

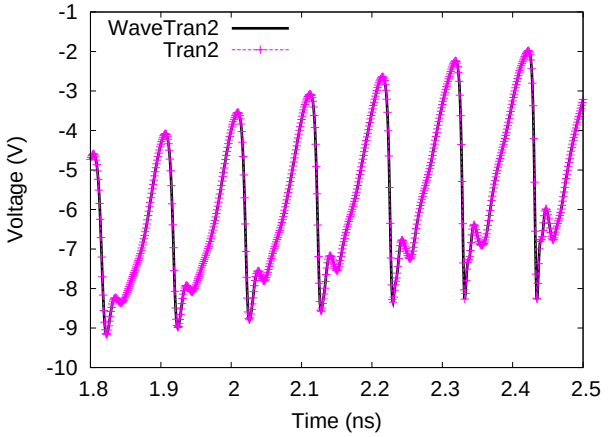

Fig. 5. Voltage at Output 5 of the Multi MMIC-Soliton-line

relaxation iteration. The reference resistance used for each simulation is shown in the table. The same value of reference resistance is used for all ports as used in Ref. [1]. In the current implementation this value is set manually as a simulation parameter. The Input Power Level row shows the AC power level of the input sources referred to 50 Ω. The Iterations rows, (*WaveTran* and *WaveTran2*) compare the number of iterations needed for *WaveTran* and *WaveTran2*. The last four rows compare the total running time using *WaveTran*, *WaveTran2*, *WaveTran2-E*, and *Tran2*. All simulated output voltages are compared with that from *Tran2* and two zoom in plots are presented in Figs. 4 and 5. As expected the results from both simulation methods agree. For all simulations on average only one Newton iteration is necessary at the nonlinear device level. Thus the computational cost of using Newton's method to obtain reflected waves from nonlinear device models formulated in terms of voltages and currents is not too expensive. Following the same trend shown in Ref. [1], *WaveTran* is slower than *Tran2* for the smaller circuit (Circuit

1) but as the circuit size increases the wave method becomes more efficient as it does not require large matrix decompositions after $\mathbf{S}$ has been calculated. It can also be observed in Table 1 that *WaveTran2-E* is always faster than *WaveTran* and even the less efficiently implemented *WaveTran2* is faster for Circuit 2.

5. CONCLUSIONS AND DISCUSSION

A modification of a novel transient analysis approach was presented in this paper. The proposed wave-based transient analysis solves the nonlinear equations using a block Newton-Jacobi algorithm. As with the original wave-based transient analysis, no matrix decomposition of a large Jacobian matrix is required at each nonlinear iteration. Thus the proposed method is also attractive to be implemented in parallel computer systems.

It was shown that the addition of Newton-Jacobi requires only a few extra computations compared to the original fixed-point approach. The simulation results presented here show that the Newton-Jacobi-based method converges faster and is more efficient for some circuits.

Although these results are encouraging some issues remain to be addressed. For example the choice of Newton-Jacobi blocks used in the current implementation may not be optimal for some circuit configurations. In circuits where two (or more) nonlinear devices are connected in parallel the coupling between them is strong and thus the two devices should be considered in the same block for optimum performance. It is also worth noting that although the block Newton-Jacobi method generally improves the convergence rate, some of the convergence properties of the original relaxation approach are lost. One of the ideas that could address this issue is to implement an adaptive algorithm that uses the Newton-Jacobi approach by default and switches to plain relaxation when convergence is poor. Another issue is the optimum selection of the reference resistance. The reference resistance has some effect on the convergence rate and the optimum value may be different for each port and also depends on the operating point. In the current implementation this resistance is the same for all ports and manually set, but in the future a better approach should be investigated.

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