AN ADAPTIVE TIME STEP CONTROL ALGORITHM FOR NONLINEAR TIME DOMAIN ENVELOPE TRANSIENT

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Abstract

In this paper we propose a new adaptive time step control algorithm for the slow time dimension of time domain envelope transient (TD-ENV) simulation. The algorithm uses two models: the first is the set of differential-algebraic equations that represent the circuit. The second is a 'coarse' model that is cheap to evaluate. The optimum time step is estimated from an error term obtained from the coarse model. The accurate model is then solved using the near optimum time step. The acceptable error for the time step estimation is adapted according to the dynamics of the system. We describe the time step control algorithm and present a case study of the transient analysis of a rectifier circuit powered by a high-frequency pulse train with a slowly-varying pulse duty cycle. The simulations show that few time steps are rejected compared with a traditional time step control algorithm.

Keywords: *Circuit simulation, Widely separated time constants, Time-domain methods*

1 INTRODUCTION

Time domain envelope transient (TD-ENV) is an envelope following method based on a bi-dimensional (or in general, multi-dimensional) representation of the time domain [1]. This method is convenient for the analysis of circuits with excitations with widely separated time scales because it requires the calculation of far fewer solution points. Consider a voltage described by the following function:

$$f(t) = \left[1 - \exp\left(-\frac{t}{\tau_1}\right)\right]\left[1 + 2\sin\left(\frac{2\pi}{\tau_2}t\right)\right],$$

were τ_1 and τ_2 are time constants. A graphical representation of this function with $\tau_1 = 25$ s and $\tau_2 = 0.1$ s is shown in Fig. 1. Many sample points are required to represent this function. For example, 2000 samples were needed in Fig. 1 for a total time of 10 s. The main idea in the TD-ENV method is to represent signals in more than one time dimension according to the scale of variation. In principle the signals are required to be periodic in at least one of the dimensions. For example, $f(t)$ is represented by the following bi-dimensional function:

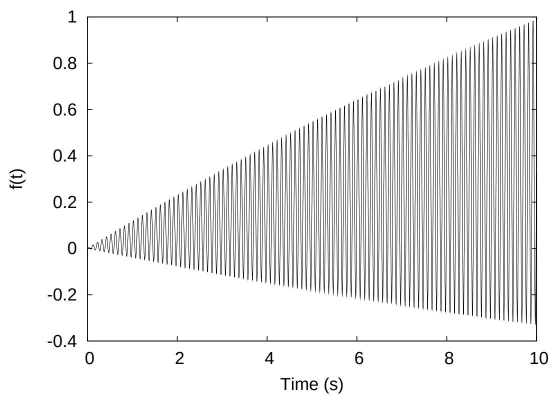

Figure 1: Plot of the function $f(t)$

$$\hat{f}(t_1, t_2) = \left[1 - \exp\left(-\frac{t_1}{\tau_1}\right)\right]\left[1 + 2\sin\left(\frac{2\pi}{\tau_2}t_2\right)\right].$$

This function is plotted in Fig. 2, for the same values of τ_1 and τ_2 used before. Only 200 sample points were necessary to represent a much longer time interval. The original function can be easily recovered by setting $t_1 = t$ and $t_2 = t$.

The system of differential-algebraic equations (DAE) that describe a circuit must be modified to use the bivariate representation. Assume a nonlinear circuit is described by a DAE of the form:

$$\dot{q}(x) + f(x) = b(t). \tag{1}$$

Here x represents the voltages and currents in the circuit, $q(x)$ the charge terms, $f(x)$ the resistive terms and $b(t)$ represents the sources. It has been proved [1] that

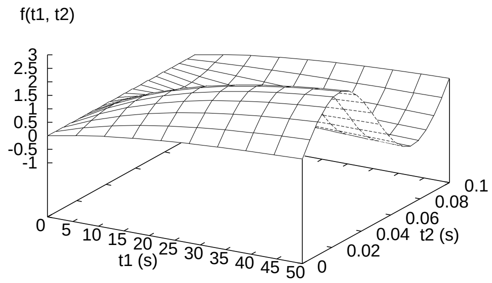

Figure 2: Plot of the function $\hat{f}(t_1, t_2)$

if $\hat{x}(t_1, t_2)$ is the solution of

$$\frac{\partial q(\hat{x})}{\partial t_1} + \frac{\partial q(\hat{x})}{\partial t_2} + f(\hat{x}) = \hat{b}(t_1, t_2), \tag{2}$$

then $x(t) = \hat{x}(t, t)$ is the solution of the system of Eq. 1. For the TD-ENV problem, the boundary conditions are:

$$\hat{x}(t_1, t_2) = \hat{x}(t_1, t_2 + T_2),$$

here, T_2 is the period of the oscillatory excitation. Now two problems must be solved: a boundary problem in the direction of t_2 and an initial condition problem in the direction of t_1. As suggested in [1], the solution of the first problem can be obtained in several ways, for example with the harmonic balance technique [2]. In this paper we have chosen a finite difference time domain (FDTD) approach. The problem in the t_1 variable can be solved with standard integration techniques such as backward Euler (BE) and trapezoidal integration. It has been reported in the literature [3,4] that the differential equations in this (t_1) direction are stiff and at times present fast variations. A variable time step is then necessary for an efficient simulation. As each step in the direction of t_1 involves the (relatively expensive) solution of a FDTD problem, it is particularly important to minimise the number of time steps in the direction of t_1. In the following section we provide an outline of a proposed time-step control algorithm that attempts to minimise the number of rejections. Then we present a case study of the transient analysis of a rectifier circuit powered by a high-frequency pulse train with a slowly-varying pulse duty cycle.

2 THE TIME-STEP CONTROL ALGORITHM

The basic idea of the algorithm is to use a coarse model of the system to predict what is the size of the optimum time step before solving the actual FDTD problem. Once the time step size is determined, the actual set of equations that represent the circuit are used to solve the FDTD problem and the truncation error is checked. The function $err(h)$ is used to estimate the optimum time step h. The pseudo-code of this function is the following:

1. Use extrapolation as a guess
2. Estimate initial residual in Newton method using coarse model
3. Return the norm of residual minus tol.

This function returns zero when the norm of the residual is exactly equal to the parameter tol. If the return value is negative, that means that the time step could be larger. The pseudo-code of the main algorithm follows:

1. $h = h_{last}$
2. if $|err(h)| < 0.2 \times tol$ then
   (a) if $err(h_{min}) > 0$ then $h = h_{min}$
   (b) if $err(h_{max}) < 0$ then $h = h_{max}$
   (c) otherwise find h such that $|err(h)| < 0.1 \times tol$
3. solve FDTD problem
4. estimate norm of normalised truncation error (E)
5. $\delta = E/E_{max}$
6. if δ small then increase tol
7. if δ large then decrease tol
8. if $h == h_{min}$ then $\delta = 1$
9. if $\delta > 1$ then reject point and go to 2
10. $h_{last} = h$
11. $t_1 = t_1 + h$
12. if not finished go to 1.

Here E_{max} is the maximum allowable value of the estimation of the normalised local truncation error (E). The parameter tol is updated at each time step according to the value of δ.

The coarse model can be obtained for example by linearising the original set of equations around the last

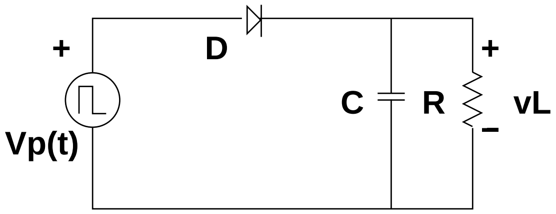

Figure 3: A rectifier powered by a variable duty-cycle pulse source.

solution point. The resulting function $err(h)$ is still nonlinear because it includes the effect of the sources. The linearised model is most useful when the nonlinear element equations are formulated in parametric form (*i.e.*, currents and voltages are expressed as functions of state variables) because frequently sharp nonlinearities, usually exponential functions, can be avoided. The value of tol can be adjusted to always provide a good initial guess to the subroutine that solves the FDTD problem. This is important to minimise the number of iterations to solve each nonlinear FDTD problem.

3 CASE STUDY

We present a case study of the transient analysis of a rectifier circuit powered by a high-frequency pulse train with a slowly-varying pulse duty cycle. The circuit is shown in Fig. 3 and is described by the following differential equations:

$$\begin{aligned} v_p(t) - v_d(x) - v_L &= 0 \\ i_d(x) - \frac{v_L}{R} - C\frac{dv_L}{dt} &= 0, \end{aligned}$$

where v_d and i_d represent the voltage and current in the diode, respectively. The state variables of the system are the load voltage v_L and the diode parameter x. The parametric diode model is described in [5]. The introduction of this state-variable based model of the diode improved the convergence of the algorithm. The equations must be now converted into PDE form,

$$\begin{aligned} v_p(t_1, t_2) - v_d(x) - v_L &= 0 \\ i_d(x) - \frac{v_L}{R} - C\left(\frac{\partial v_L}{\partial t_1} + \frac{\partial v_L}{\partial t_2}\right) &= 0. \end{aligned}$$

Now the equations are discretised using the BE formula:

$$\begin{aligned} v_p(t_1^{n,m}, t_2^{n,m}) - v_d^{n,m} - v_L^{n,m} &= 0 \\ i_d^{n,m} - v_L^{n,m}/R & \\ -C(v_L^{n,m} - v_L^{n-1,m})/h_1 & \\ -C(v_L^{n,m} - v_L^{n,m-1})/h_2 &= 0, \end{aligned}$$

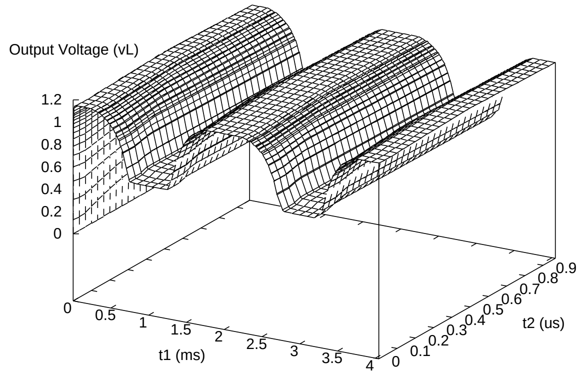

Figure 4: Bidimensional plot of the load voltage (v_L)

where n and m represent the index value in the t_1 and t_2 directions, respectively. The periodic boundary conditions in the t_2 direction are:

$$v_L^{n,-1} = v_L^{n,m_{max}}.$$

In this example the coarse model for index n is made by linearising the diode model around all m solution points at the previous point in the t_1 direction ($n-1$):

$$\begin{aligned} i_d^{n,m} &\approx i_d^{n-1,m} + \frac{\partial i_d^{n-1,m}}{\partial x}\Delta x^{n,m} \\ v_d^{n,m} &\approx v_d^{n-1,m} + \frac{\partial v_d^{n-1,m}}{\partial x}\Delta x^{n,m}, \end{aligned}$$

where $\Delta x^{n,m}$ is the increment in the value of the state variable. The required derivatives are already available from the nonlinear solution of the previous point.

The TD-ENV method to simulate this circuit was implemented in the Octave [6] program. The following parameters were used: $R = 2$ kΩ, $C = 10$ nF, diode saturation current: 1 fA, diode series resistance: 100 Ω, peak pulse level: 2 V, period: 1 μs and duty cycle: $0.3 + 0.2c$, where c is a factor that varies between $+1$ and -1 with a period of 2 ms. The output voltage and the diode parameter are shown in Figures 4 and 5, respectively. It can be observed that the circuit has a strong nonlinear behaviour. Fig. 6 shows the variation of the time step (h). The size of the time step is maximum during the times the duty cycle is constant and is reduced when the duty cycle varies. For comparison purposes, a simulation using a more traditional time-step control algorithm [7] was also performed. In that algorithm, only the truncation error is used to determine the next time step size. The stop time, number of grid points in the t_2 direction and tolerances for both simulations are the same. Table 1 summarises the results of both simulations. It is observed that the proposed algorithm requires less than half the number of FDTD solutions.

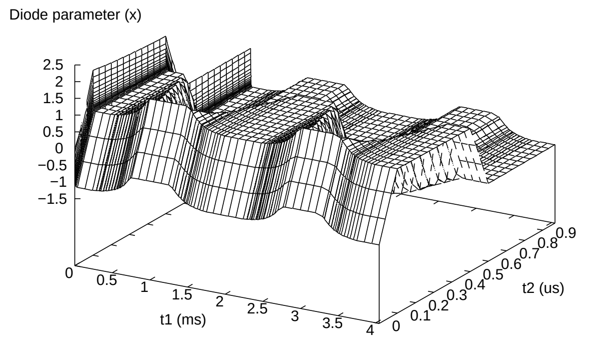

Figure 5: Bidimensional plot of the diode parameter (x)

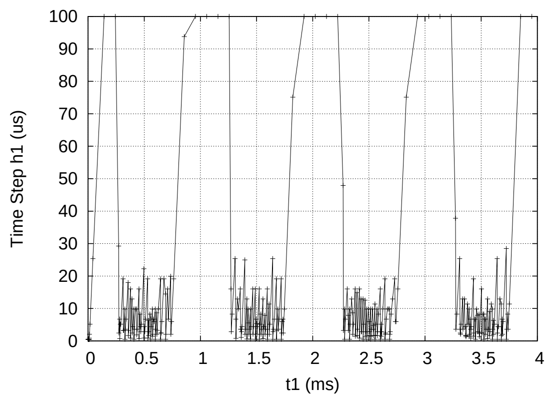

Figure 6: Time step in the direction of t_1

4 CONCLUSIONS

We have implemented a time-step control method to simulate circuits in the time domain using the MPDE approach, enabling strong nonlinearities to be handled effectively. This technique represents a system of differential equations with widely separated time scales efficiently by using multivariate functions. This leads to a partial differential form for the system equations. Computation and memory are independent of the separation between time scales, leading to considerable savings when the disparity is large. This technique is applicable to circuits with widely separated time scales, that are difficult or impossible to simulate with previous techniques.

The simulation results show that with the new method fewer time steps are rejected compared with a traditional time step control algorithm. Also, the time step is chosen such that the residual of the initial guess in the Newton method is lower than some tolerance. This helps to reduce the number of Newton iterations necessary. This is particularly important with TD-ENV simulations because each step in the slow direction involves a relatively expensive solution of a nonlinear boundary-type problem (FDTD) in the fast time dimension. We have achieved these results by extending the techniques used for transient analysis of circuits to a bi-dimensional problem. More research is necessary to see if any technique used for the numerical solution of PDE may applicable in this kind of problem.

	Accepted	Rejected	Total
Proposed	354	76	430
Traditional	581	324	905

Table 1: Comparison of the number of time steps with the proposed and traditional time-step control methods

Acknowledgements

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