Fully Physical Coupled Electro-Thermal Modelling of Power Devices and Circuits

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Abstract— **The compact Leeds thermal impedance matrix model of time-dependent, non linear heat flow, in complex 3-dimensional volumes is introduced in the context of coupled electro-thermal CAD. Transient co-simulation, based on implementation in a microwave circuit simulator, Transim (NCSU), is developed by introduction of a time-stepping scheme with repeated analytical resetting of initial conditions for distributed thermal sub-volumes. This thermal impedance matrix model is compared against the electro-thermal modelling capability of programme SABER, as described by Hefner and Blackburn. An explicit example of global coupled EM-electro-thermal simulation for a large microwave subsystem is described.**

Keywords: thermal, electro-thermal, simulation, circuits

I. Introduction

The Leeds thermal impedance matrix model has been described at length in [1]. This compact model is obtained by fully analytical solution of the heat diffusion equation in complex structures, based on domain decomposition into regular sub-volumes. The non linear heat diffusion equation is converted into a fully linear equation by appropriate transformations of variable [2]–[4]. Analytical series solutions for thermal subsystems are accelerated for rapid precomputation prior to coupled electro-thermal co-simulation [5]. The thermal model is coupled to both a fully physical device simulator, the quasi-2-dimensional Leeds Physical Model of MESFETs and HEMTs [6], [7], and to a microwave circuit simulator, Transim (NCSU) [1]. This gives the capability for fully self-consistent electro-thermal simulation, on CAD timescales, in a wide range of devices and circuits, from microwave MESFETs and HEMTs, through high power IGBTs, to MMICs, MCMs and whole spatial power combining MMIC arrays. The complexity of the electro-thermal simulation problem for large microwave subsystems is illustrated in Fig. 1. The aim is to provide a thermal description from sub-micron active device channels, to the ∼10 cm × ∼10 cm scale of the whole grid array, including the effects of all structure from power FET surface metallisation, air bridges and vias, through to the

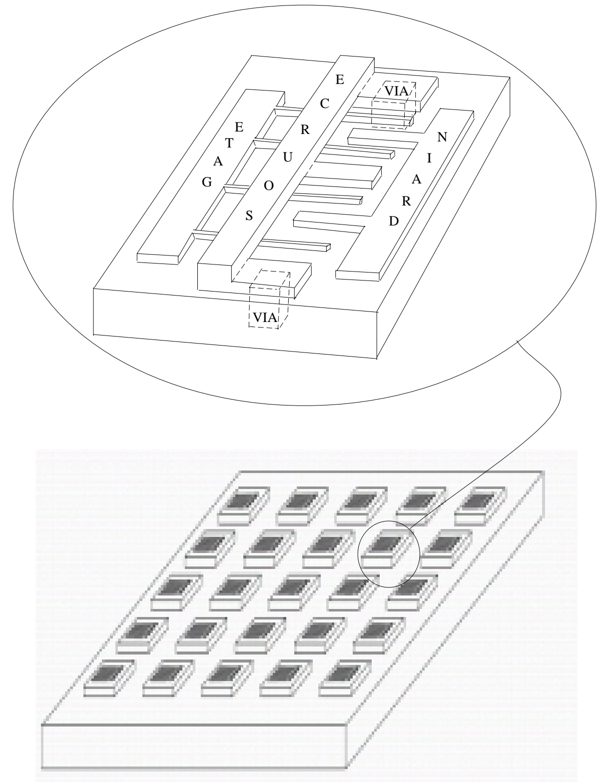

Fig. 1. **Schematic (not to scale) of a power FET grid array representative of one form of spatial power combiner architecture.**

$N \times N$ GaAs die on substrate array. This thermal description must be compatible with coupled electro-thermal co-simulation of the whole device on CAD timescales.

This paper describes a development of the transient electro-thermal co-simulation based on the Leeds thermal impedance matrix approach. This time-stepping approach, with analytical resetting of initial conditions for the distributed thermal system, offers significant improvement in the speed of electro-thermal co-simulation. The authors have previously compared the Leeds thermal impedance matrix model against the comprehensive electro-thermal modelling and measurement capability of

Szekely *et al.* [8]–[10], and the Leeds model has been clearly placed in the context of wider thermal and electro-thermal modelling in [1]. In this paper, its advantages are illustrated by comparison against the well known electro-thermal modelling capability of programme SABER, as described by Hefner and Blackburn [11], [12]. Finally, an explicit example of global coupled EM-electro-thermal simulation of a large microwave subsystem is presented [13].

II. **Thermal Model**

Under successive Kirchhoff transformation of temperature, $T \to \theta$, and transformation of time, $t \to \tau$ [2]–[4], the heat diffusion equation,

$$\nabla.\left[\kappa(T)\nabla T\right] + g(x,y,z,t) = \rho C \frac{\partial T}{\partial t}, \qquad (1)$$

becomes,

$$\nabla^2\theta + \frac{g}{\kappa_S} = \frac{1}{k_S}\frac{\partial \theta}{\partial \tau}. \qquad (2)$$

To construct the time dependent thermal solution with volume heat sources/sinks and arbitrary initial conditions, requires the solution of Helmholtz's equation in Laplace transform s-space. Multi-layer systems, Fig. 2, are treated fully analytically by use of a 2×2 transfer matrix approach. Arbitrary N-level structures can be treated [1], [14].

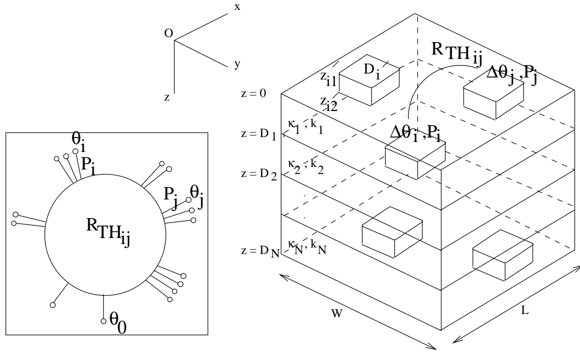

Fig. 2. N-level multilayer with arbitrarily distributed volume sources for fully analytical construction of thermal impedance matrix $R_{TH_{ij}}(s)$. Inset: N-port described by generalised multiport thermal Z-parameters, $R_{TH_{ij}}(s)$.

By averaging the resulting analytical solution over power dissipating and temperature sensitive volumes and areas, the solution reduces to the thermal impedance matrix form,

$$\overline{\Delta\theta_i} = \sum_j R_{TH_{ij}}(s)\overline{P_j}, \qquad (3)$$

where $\overline{\Delta\theta_i}$ is the Laplace transformed temperature rise of element i above its initial temperature, $R_{TH_{ij}}(s)$ is the thermal impedance matrix in Laplace s-space and the $\overline{P_j}$ are the transformed time-dependent fluxes due to power dissipation in elements, $j = 1, ..., i, ...M$.

$R_{TH_{ij}}(s)$ then represents the generalised multi-port thermal Z-parameters for a thermal N-port, Fig. 2 (inset), with the Z-parameters evaluated either directly in frequency space, $s \to j\omega$, giving the impulse response, $R_{TH_{ij}}(j\omega)$, or by analytical or numerical Laplace inversion in the time-domain, giving the step response, $R_{TH_{ij}}(\tau) = \mathcal{L}^{-1}\left\{R_{TH_{ij}}(s)\frac{1}{s}\right\}$ [1].

III. **Coupled Electro-Thermal Transient**

The thermal impedance matrix in s-space can be used directly in coupled electro-thermal harmonic balance (HB) simulations. In this case, the matrix of frequency dependent complex phasors corresponds to the network parameters of the distributed multi-port thermal network. It is inserted directly into the modified nodal admittance matrix (MNAM) for the microwave system and so does not increase the number of non linear equations describing the coupled solution.

In the coupled electro-thermal transient problem, Laplace transformed active power dissipations, $\overline{P}_j(s)$, are not known explicitly and must be obtained by self-consistent solution. To combine the electrical and thermal descriptions, the corresponding $P_j(\tau)$ must therefore be discretised in time. Dividing the time interval of interest into equal subintervals of length $\delta\tau$, with the $P_j(\tau)$ taking the piecewise constant form (for illustration)

$$P_j(\tau) = P_j^{(n)} \;\text{ for }\; (n-1)\delta\tau < \tau \leq n\delta\tau, n = 1, ..., N \qquad (4)$$

then gives

$$\overline{P}_j(s) = \sum_n \frac{1}{s}(1 - e^{-s\delta\tau})e^{-(n-1)s\delta\tau}P_j^{(n)}. \qquad (5)$$

Laplace inverting the impedance matrix equation, Eq (3), the temperature rise of element i at time $\tau = m\delta\tau$, $\Delta\theta_i^{(m)}$, is obtained as a function of the $P_j^{(n)}$. Writing $\Delta\theta_i^{(m)} = \Delta\theta_i^{(m)}(P_i^{(m)})$ from the electrical model then gives,

$$\Delta\theta_i^{(m)}(P_i^{(m)}) = \mathcal{L}^{-1}\left\{R_{TH_{ij}}(s)\overline{P}_j(s)\right\}_{\tau = m\delta\tau}, \qquad (6)$$

$$= \sum_n \sum_j \left[u(m-n+1)R_{TH_{ij}}((m-n+1)\delta\tau) - u(m-n)R_{TH_{ij}}((m-n)\delta\tau)\right]P_j^{(n)}, \qquad (7)$$

where $u(\tau)$ is the unit step function.

This corresponds to N systems of equations in M unknowns, where N is the number of discretised time points in the time interval under consideration, and M is the number of power dissipating or temperature sensitive elements. The Laplace inversion, with piecewise constant power dissipation, avoids any explicit convolution operation.

The entire thermal description can therefore be obtained by precomputation of $R_{TH_{ij}}(\tau)$ at timesteps, $\tau = n\delta\tau$, $n = 0, ..., N$. These precomputed values can be stored for repeated re-use in different electro-thermal simulations. For reduction of precomputation time, the $R_{TH_{ij}}(\tau)$ can be generated at intervals, and interior points obtained accurately by interpolation. This is a time-domain approach equivalent to representation of a frequency space transfer function by a rational polynomial fit.

IV. Arbitrary initial conditions

To treat the case of arbitrary initial conditions, necessary for a time-stepping coupled electro-thermal solution with repeated resetting of initial conditions, solution of Eq. (2) is required with the same adiabatic side face boundary conditions and with the radiation boundary condition on $z = 0, D$ given by

$$\alpha_{0,D}\kappa_S \frac{\partial \overline{\theta}}{\partial z} + H_{0,D}\left(\overline{\theta} - \overline{\theta}_{0,D}(x,y;s)\right) + \overline{p}_{0,D}(x,y;s) = 0. \quad (8)$$

To solve Eq. (2) for arbitrary initial temperature distribution, $\theta(t = 0)$, write,

$$\overline{\theta} = \overline{\theta}_h + \overline{\theta}_s, \quad (9)$$

where $\overline{\theta}_h$ satisfies the homogeneous equation

$$\nabla^2 \overline{\theta}_h - \frac{s}{k}\overline{\theta}_h = 0, \quad (10)$$

with adiabatic side wall boundary conditions, and radiation boundary conditions on $z = 0, D$,

$$\alpha_{0,D}\kappa_S \frac{\partial \overline{\theta}_h}{\partial z} + H_{0,D}\left(\overline{\theta}_h - \overline{\theta}_{0,D}(x,y;s)\right) + \overline{p}_{0,D}(x,y;s) = 0. \quad (11)$$

Then $\overline{\theta}_s$ satisfies the equation with sink term $-\theta(t = 0)/k$,

$$\nabla^2 \overline{\theta}_s - \frac{s}{k}\overline{\theta}_s = -\frac{1}{k}\theta(t = 0), \quad (12)$$

with adiabatic side wall boundary conditions, and radiation boundary conditions on $z = 0, D$,

$$\alpha_{0,D}\kappa_S \frac{\partial \overline{\theta}_s}{\partial z} + H_{0,D}\overline{\theta}_s = 0. \quad (13)$$

It is important to note that this boundary condition on the inhomogeneous problem is independent of both power density $\overline{p}_{0,D}(x, y; s)$ and temperature $\overline{\theta}_{0,D}(x, y; s)$ on top and bottom faces, $z = 0, D$.

The solution for $\overline{\theta}_h$ of Eq. (10) is of the same separation of variables form as given in [1]. The solution for $\overline{\theta}$ can then be written

$$\overline{\theta}(s) = \sum_{mn} \cos\lambda_m x \cos\mu_n y \times \left(Z_{mn}(z) + C_{mn}\cosh\gamma_{mn}z + S_{mn}\sinh\gamma_{mn}z\right), \quad (14)$$

where,

$$Z_{mn}(z) = Z_{mn}|_0 \cosh\gamma_{mn}z + \frac{dZ_{mn}}{dz}|_0 \frac{1}{\gamma_{mn}}\sinh\gamma_{mn}z - \frac{e^{\gamma_{mn}z}}{k}\int_0^z e^{-2\gamma_{mn}z'}\int_0^{z'} e^{\gamma_{mn}z''}\theta_{mn}(z'')dz''dz', \quad (15)$$

with

$$\theta_{mn}(z) = \frac{4}{(1+\delta_{m0})(1+\delta_{n0})\kappa L W} \times \int_0^L\int_0^W \cos\lambda_m x \cos\mu_n y\, \theta(t = 0)\, dy dx. \quad (16)$$

Putting $\alpha_0 = 1, H_0 = 0, \alpha_D = 0$ and $H_D = 1$ in Eq. (13), corresponding to a specified flux boundary condition on $z = 0$ and a specified temperature boundary condition on $z = D$, boundary conditions on the Z_{mn} become,

$$\frac{dZ_{mn}}{dz}|_0 = 0, \quad Z_{mn}|_D = 0. \quad (17)$$

Imposing these boundary conditions, the full solution for $\overline{\theta}$, Eq. (9), with arbitrary initial temperature distribution, $\theta(t = 0)$, is obtained. Eqns. (14)–(16) above provide an explicit double Fourier series construction for the time-dependent temperature in a MMIC with arbitrary initial temperature distribution.

To treat the specific case of a temperature distribution obtained at step, $r + 1$, from initial conditions obtained as the result of a series of heat diffusion problems, $r, r - 1, ..., 0$, Eqns. (14)–(16) can be written

$$Z_{mn}^{(r+1)} = \hat{O}^{r+1}\left\{Z_{mn}^{(0)}(z)\right\} + \sum_{i=1}^{r+1}\hat{O}^i\left\{C_{mn}^{(r+1-i)}\cosh\gamma_{mn}z + S_{mn}^{(r+1-i)}\sinh\gamma_{mn}z\right\} \quad (18)$$

where superscripts $(r), (r + 1)$ distinguish Z_{mn}, C_{mn}, S_{mn} corresponding to consecutive time ranges, i.e. the solution of calculation, r, provides the initial conditions for calculation, $r + 1$. The $C_{mn}^{(r+1)}, S_{mn}^{(r+1)}$ are given immediately by solution of the homogeneous equation, Eq. (10) [1], and operator $\hat{O}$ is defined by

$$\hat{O}\{f(z;s)\} = \frac{\cosh\gamma_{mn}z}{\cosh\gamma_{mn}D}\frac{e^{\gamma_{mn}D}}{k}\int_0^D e^{-2\gamma_{mn}z'}\int_0^{z'} e^{\gamma_{mn}z''} \times \mathcal{L}^{-1}\{f(z'';s)\}_{t=T}\, dz''dz' - \frac{e^{\gamma_{mn}z}}{k}\int_0^z e^{-2\gamma_{mn}z'}\int_0^{z'} e^{\gamma_{mn}z''} \times \mathcal{L}^{-1}\{f(z'';s)\}_{t=T}\, dz''dz'. \quad (19)$$

As a concrete example, initial temperature distribution in calculation $r = 0$, can be assumed to be

uniform and equal to ambient. Assuming adiabatic surface boundary conditions, with heatsink mounting at ambient temperature, then gives $Z^0_{mn}(z)$ and $C^{(r)}_{mn}, S^{(r)}_{mn}$ for all r. The $\overline{P}_i$ of Eq. (3) in the $C^{(r)}_{mn}, S^{(r)}_{mn}$ are given by,

$$\overline{P}^{(r)}_i = \sum_{\nu=0}^{N} c_\nu(s) P^{(r)}_{i\nu}, \qquad (20)$$

where the $P^{(r)}_{i\nu}$ are fixed values of the power dissipation of each active device, i, evaluated at the νth instant within the rth time range, 0 to τ. The coefficients $c_\nu(s)$ are uniquely determined by the order of interpolation, N, assumed between instantaneous power densities, $P^{(r)}_{i\nu}$.

To make the repeated z', z'' integrations of Eqns. (18) and (19) easily tractable analytically after Laplace inversion, the inversion is performed numerically [15], [16],

$$\mathcal{L}^{-1}\left\{f(z;s)\right\}_{t=\tau} = \sum_\mu w_\mu f(z; s_\mu), \qquad (21)$$

with w_μ and s_μ determined uniquely for a given τ. Explicit construction of $Z^{(r+1)}_{mn}$ given by Eq. (18) is then immediate.

To obtain active device temperatures, $\overline{\theta}^{(r+1)}_{av_i}$, surface averages are constructed, giving finally the thermal impedance matrix equation, Eq. (3), in the form,

$$\overline{\theta}^{(r+1)}_{av_i} = \sum_{ij} R_{TH_{ij}}(s)\overline{P}^{(r+1)}_j + \sum_{mn} \frac{I^i_{mn}}{I^i_{00}} Z^{(r+1)}_{mn}|_0. \qquad (22)$$

Thermal updates in the coupled electro-thermal problem are then determined jointly by multiplication of impedance matrix, $R_{TH_{ij}}(s)$, by power densities, $\overline{P}^{(r+1)}_j$, with initial temperature information for the rest of the distributed system updated by $Z^{(r+1)}_{mn}$. The temperature variation with time has thus been written as the sum of a variation forced by applied power dissipation, and a term due to the tendency towards uniform temperature, in the absence of applied power, when starting from an inhomogeneous initial temperature distribution. It should be noted that the $Z^{(r+1)}_{mn}$ depend only on $\overline{P}^{(s)}_j$ for $s \leq r$. Therefore, the significance of the result, Eq. (22), is that temperature updates at the active device elements, Eq. (3), are decoupled from updates for the rest of the distributed thermal system, Eqns. (18) and (22). The self-consistent electro-thermal problem is therefore minimal and dictated only by the (small) number of terminals of the thermal N-port. Also, in a time-stepping solution with uniform time steps, the $\sim N^2/2$ distinct elements of $R_{TH_{ij}}(s)$ only have to be evaluated at a single instant, τ. This avoids the need to sum the effects of all previous power steps, at each self-consistent non linear iteration, thus accelerating the coupled electro-thermal co-simulation. In fact the operations count which is strictly $O(M^2)$ for M timesteps in the original transient simulation, becomes only $O(M)$. This saving is partially offset by the fixed operations cost needed at each time step, to reset the initial conditions for the rest of the distributed thermal system.

This solution should be compared against more conventional generation of a lumped element RC-network approximation for the distributed thermal system. No model reduction is implemented in the above approach, beyond that implicit in use of numerical Laplace inversion. The thermal network represented by $R_{TH_{ij}}(s)$ only has to be constructed once (matrix elements evaluated at just one time point, τ). Detailed comparisons of precomputation costs and coupled electro-thermal co-simulation costs, for the $R_{TH_{ij}}(\tau)$ approach of Section III, the above approach, and more conventional SPICE-like circuit simulations, based on a variety of RC-network reduction models, will be presented elsewhere.

V. Comparison with SABER

The electro-thermal capability in SABER, as described by Hefner and Blackburn [11], [12], was developed initially for power electronic applications, i.e. power electronic circuits, power modules and power circuit boards, containing power diodes and power insulated gate bipolar transistors (IGBTs) operating at, say, 600 V and 25 A. These papers state that the approach is applicable to electro-thermal systems in general.

The Leeds thermal impedance matrix model in Transim (NCSU) offers a ready extension to other electro-thermal systems, especially microwave and millimeter-wave systems, though it can treat power electronics and IGBTs as well. As a microwave circuit simulator, Transim also provides the capability for integral EM modelling, and can treat complicated structures like MMIC array spatial power combiners, Fig. 1, with complicated, 3-dimensional, non linear, heat flows.

An advantage of Transim over SABER is that Transim has harmonic balance capability. The SABER thermal capability that has been described in the literature is purely transient, i.e. it solves only the time-domain heat diffusion equation, not the frequency domain, or Laplace transform s-space (complex frequency space) case. It cannot therefore readily treat problems like non linear distortion, spectral regrowth or ACPR, central to study of microwave and millimeter-wave systems. Quite generally, although most circuit simulation programmes focus on transient analysis, it is the steady-state behaviour of analogue and microwave circuits which is typically of interest to a designer [17].

Hefner and Blackburn describe only a small number of generic thermal component types: Si chip, package, module and heatsink. Each of these elements requires customised thermal analysis before entry into the SABER component library. In contrast, the most primitive thermal subelements in Transim are simple rectangular subvolumes and multilayers, which can be connected together at will, by netlist specification, to describe any arbitrarily complex 3-d structure. The user does not need to do any heatflow analysis in Transim, this is done essentially exactly by the thermal impedance matrix approach. All the user has to do is connect subvolumes. However, once a complex structure like a metallised FET, MMIC, MCM *etc.*, has been built, its corresponding thermal impedance matrix can be saved for later re-use, i.e. it becomes a library component.

In SABER, a Si chip has a single surface heating area or volume (though a module can have multiple heat surface heat sources or volumes). In contrast, in Transim, elementary thermal elements can have arbitrary distributions of volume power dissipations located anywhere in the body of the thermal subvolume.

SABER works implicitly on the assumption of vertical heat flow, from top Si chip to bottom heat sink. It allows only vertical matching of its thermal components. In contrast, thermal elements in Transim can be connected vertically, horizontally and embedded within each other. This give much greater flexibility for describing complex heat flows. In particular, it is not easy to see how SABER would ever allow accurate description of a fully 3-d heat flow such as that occurring in a tray/card spatial power combiner.

Related to this, SABER approximates 3-d heat flows by assuming (approximate) symmetry (rectangular, cylindrical or spherical) and so reducing the corresponding 3-d heat diffusion equation to a 1-d form. It then takes an approach of *ad hoc* modifications to treat effects due to lateral and spherical heat spreading and the effects of 3-d geometry at edges and corners. Importantly, it solves the reduced 1-d equation only approximately based on finite difference discretisation. In contrast, the thermal impedance matrix approach in Transim solves the 3-d heat diffusion equation exactly analytically in all its finite rectangular subvolumes, i.e. no 1-d approximations or edge effects.

To treat non linearity of the heat diffusion equation, arising from the temperature dependence of Si material parameters, SABER integrates the discretised 1-d equation. It effectively assumes that the temperature dependent material parameters are uniform within a given discretised layer. This is only an approximation. It's validity for the approximate SABER description of complex 3-d heat flow is not at all obvious. In contrast, the Kirchhoff and time variable transformations in Transim treat the non linearity 'nearly exactly' in full 3-d.

A major advantage of the Leeds thermal impedance matrix model in Transim, over SABER, is that SABER has no facility to treat structural fine detail, such as surface metallisation, air bridges, vias, *etc.*, that are known to be essential in accurate description of power FETs and MMICs, *e.g.*, [18]. The thermal impedance matrix method in Transim can describe such detail in a minimally compact manner, based on analytical, accelerated series, solutions which remove all resolution limitations on solutions over length scales from submicron device channels, to ∼10cm structures.

SABER uses a logarithmic grid in its 1-d finite difference discretisation, to reduce the number of internal nodes far below that of a typical 3-d finite difference discretisation. However, the number of internal nodes is still significant. In contrast, the thermal impedance matrix description in Transim is minimal, i.e. it contains no redundant internal nodes, and only contains the minimum number of nodes consistent with connection of the electrical and thermal circuits. It achieves this minimal description whilst still incorporating full details of fine structure on CAD timescales.

SABER already has some library elements constructed for it. However, the boundary condition independent (BCI), compact thermal package libraries being developed on the DELPHI/SEED project (supported by a number of major industrial concerns) [19], will all be fully compatible with the thermal impedance matrix model in Transim.

Finally, Transim now also has the fully physical Leeds Physical Model of MESFETs and HEMTs coupled to it, which will give it a unique capability not matched by the (physics based) extracted models in SABER.

VI. Results

This paper describes the first application of the Leeds thermal impedance matrix in Transim, to the global coupled EM-electrical-thermal simulation of a 'tray/card' spatial power combining array, Fig. 3, and represents the highest level of global modeling reported to date for such a structure.

Fig. 4 shows the results of a coupled EM-electro-thermal, single-tone HB simulation of the spatial power combining array. The figure illustrates the impact of thermal effects on total system output voltage waveform at the output waveguide.

VII. Conclusions

The Leeds thermal impedance matrix model has been introduced. Transient simulation based on this model has been further developed by means of repeated analytical resetting of initial conditions, promising significant reductions in coupled electro-thermal co-simulation CPU times. To illustrate the

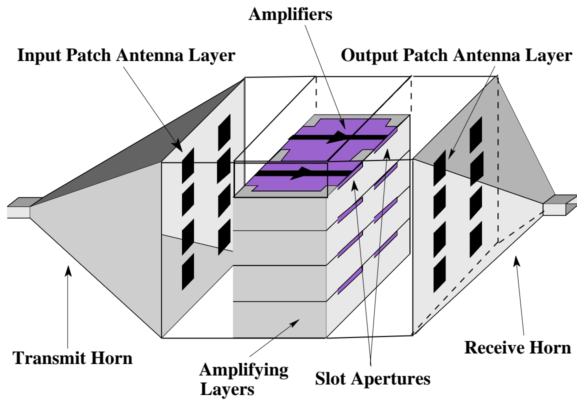

Fig. 3. **X-band 'tray/card' MMIC array spatial power combiner used for global EM-electro-thermal simulation and experimental validation.**

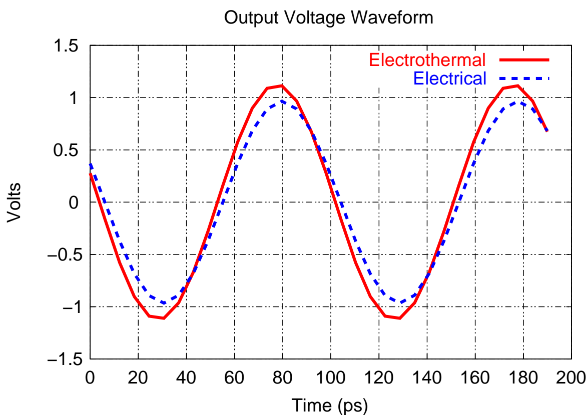

Fig. 4. **Coupled EM-electro-thermal, single-tone HB simulation of the spatial power combining MMIC array, illustrating thermal effects on total system output.**

advantages of the Leeds thermal impedance matrix approach, implemented in a microwave circuit simulator, Transim (NCSU), the coupled model has been compared against the electro-thermal simulation programme SABER. Finally, an example has been presented of electro-thermal simulation of a large microwave system.

VIII. Acknowledgement

This work was supported by the U.S. Army Research Office through Clemson University as a Multidisciplinary Research Initiative on Quasi-Optics, agreement Number DAAG55-97-K-0132.

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