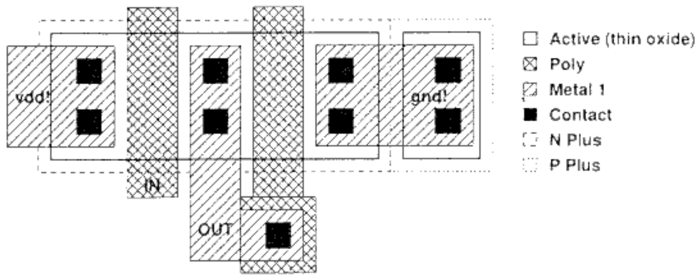
ENGI 5131
Midterm Exam — Winter 2017

Parameters for all problems: $I_{SQn} = 280$ nA, $V_{thn} = 0.5$ V, $|\partial L/\partial V_{DS}|_n = 0.03$ μm/V, $I_{SQp} = 80$ nA, $V_{thp} = -0.55$ V, $|\partial L/\partial V_{DS}|_p = 0.04$ μm/V, $n_n = n_p = 1.3$, $t_{OX} = 6$ nm and $A_\beta = 3$ %μm.

Note: show units for all results.

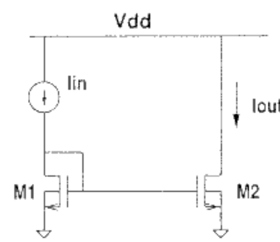
1. The following layout is for a process with p-type substrate. Draw an schematic diagram of the circuit in this layout. Use the provided node labels.



2. In the figure, $I_{in} = 20$ μA, $V_{DD} = 2.5$ V and $(W/L)_1 = (W/L)_2 = (2\mu\text{m}/2\mu\text{m})$.

   (a) Calculate I_{out} considering the effect of channel-length modulation, assuming perfect matching. Show calculation steps.

   (b) Neglecting channel-length modulation, estimate the standard deviation in output current due to transistor mismatch.



3. A standard cascode PMOS current mirror is designed to operate at an inversion level of 30. The supply voltage is 3 V.

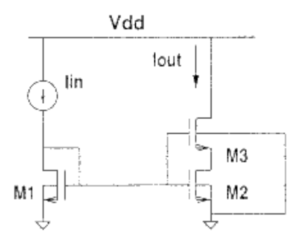
   (a) What is the maximum output voltage (at the load) that the mirror can handle?

   (b) If the design is changed to a high-swing cascode (with same inversion level), what is the maximum output voltage that the mirror can handle?

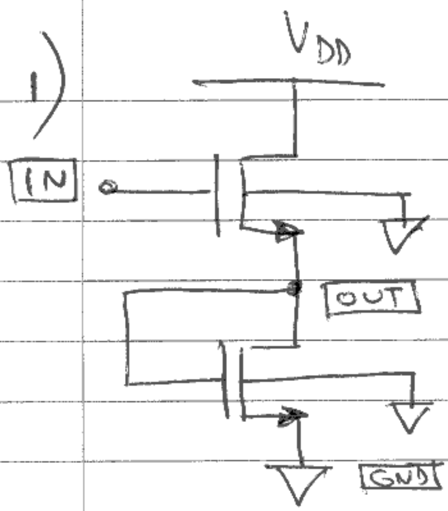
4. In the figure, $I_{in} = 0.5$ μA, $V_{DD} = 3$ V and all transistors are equal with $(W/L) = 20$. Justify your answers.

   (a) Specify regions of operation for each of M1, M2 and M3: inversion level (weak, moderate or strong) and triode or active.

   (b) Neglecting channel-length modulation calculate i_{f2} and i_{r2}.



1)

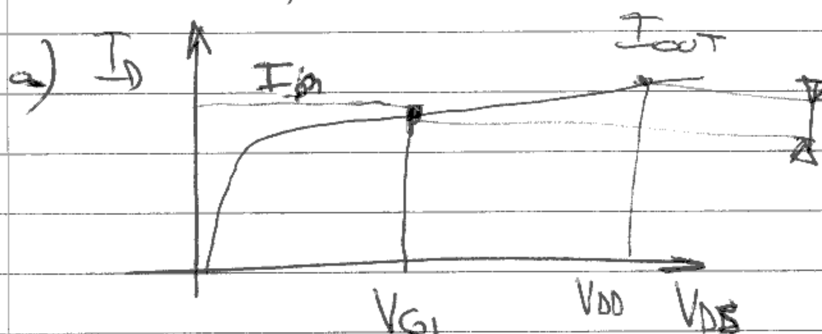


(+3) { CLEAR LABELS CORRECT

---

2) $I_{in} = 20\mu A$ $\quad V_{DD} = 2.5V$ $\quad \frac{W}{L} = \frac{2\mu m}{2\mu m}$

a)



$$\Delta I_D = \frac{V_{DD} - V_{G1}}{r_o} \quad (+1)$$

$$V_{G1} = V_{th} + nV_T\, f(i_F)$$

$$i_F = \frac{20\mu A}{I_{SQ}\frac{W}{L}} = 71.4 \qquad \approx \sqrt{71.4}$$

$$V_{G1} = 0.5V + 1.3 \times 26mV \times 8.52 = 0.79V \quad (+1)$$

$$I_{OUT} = I_{in} + \frac{V_{DD} - V_{G1}}{r_o} = I_{in} + \frac{V_{DD} - V_{G1}}{V_A} \times I_{in}$$

$$= I_{in}\left(1 + \frac{V_{DD} - V_{G1}}{V_A}\right)$$

$$V_A = \frac{L}{\left|\frac{\partial L}{\partial V_{DS}}\right|} = \frac{2\mu m}{0.03\frac{\mu m}{V}} = 66.7V \quad (+1)$$

$$\therefore I_{OUT} = 20\mu A\left(1 + 0.026\right)$$

$$\cong 20.5\,\mu A$$ (+1)

b)

$$\sigma\left(\frac{\Delta I_D}{I_D}\right)^2 = \frac{2}{4(\mu m)^2}\left[\underbrace{\underbrace{\left(\frac{6\,mV\,\mu m}{1.3\times 26\,mV}\right)^2}_{0.032}\underbrace{\frac{\ln(1+71.4)}{71.4}}_{0.059}}_{0.0019} + \underbrace{\left(0.03\,\mu m\right)^2}_{0.0009}\right]$$ (+1)

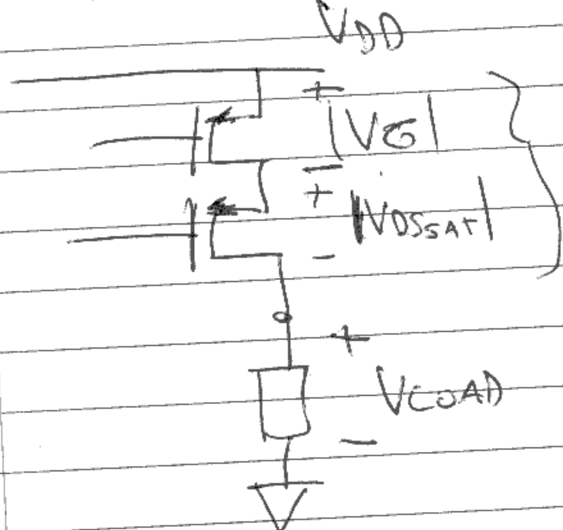
$$= 0.014$$ (+1)

$$\therefore \sigma\left(\frac{\Delta I_D}{I_D}\right) = \sqrt{0.014} = 0.037 = 3.7\,\%$$ (+1)

---

3) $i_f = 30 \qquad V_{DD} = 3V$ (PMOS CASCODE MIRROR)

a)



MINIMUM VOLTAGE DROP

(+1) CONSIDER VDD

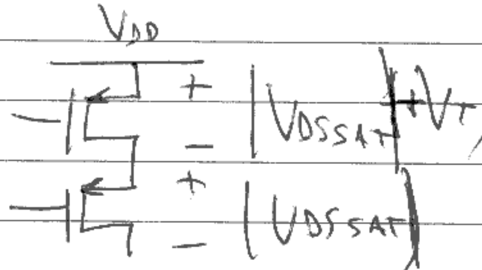
$$\therefore V_{LOAD\,MAX} = V_{DD} - |V_{DSSAT}| - |V_G|$$

$$|V_G| = |V_{thp}| + nV_T\,\overbrace{F(i_f)}^{5.1} = 0.722V$$ (+1)

$$|V_{DSSAT}| = V_T\left(3 + \underbrace{\sqrt{1+i_f}}_{5.56}\right) = 0{,}223\,V$$

$$\boxed{V_{LOAD\,MAX} \cong 2{,}05\,V} \quad (+1)$$

b)



$$\therefore V_{LOAD\,MAX} = V_{DD} - \underbrace{2|V_{DSSAT}| - V_T}$$

$$= V_T\left(6 + 2\sqrt{1+i_f} + 1\right)$$

$$\boxed{V_{LOAD\,MAX} = V_{DD} - V_T\left(7 + 2\sqrt{1+i_f}\right) \cong 2.53\,V} \quad (+1)$$

---

4) M_1: DIODE-CONNECTED → ACTIVE (+2) M_1, M_3

$$i_{f1} = \frac{0.5\mu A}{20\, I_{SQ}} = 0.089 \Rightarrow \text{WEAK INVERSION}$$

(WE SHOULD ACTUALLY VERIFY THAT $V_G > V_{DSSAT}$ TO BE SURE OF ACTIVE REGION) (+1) BONUS

$$V_{G1} = 0.5V + n V_T \mathcal{F}(0.089) = 1{,}69\,V > 100\,mV \text{ (OK)}$$

M2: $\dfrac{V_G - V_{th}}{n} = V_P$ SAME AS $M_1 \Rightarrow$ WEAK INVERSION

$$\frac{V_P - 0}{V_T} = \mathcal{F}(i_{f2}) \qquad \frac{V_P - V_{D2}}{V_T} = \mathcal{F}(i_{r2}) \quad (1)$$

$(i_{f2} = i_{f1})$ CAN NOT TELL IF ACTIVE OR NOT YET.

(+1)

M3: $V_{P_3} = V_{P_2} = V_{P_1} \Rightarrow$ ALL WEAK INVERSION

$$\frac{V_P - V_{S3}}{V_T} = \mathcal{F}(i_{f3}) \quad (2)$$

M3

. SINCE $V_{D3} = V_{DD} = 3V \Rightarrow V_{DS_3}$ LARGE $\Rightarrow$ ACTIVE

$$I_{OUT} = I_S \, i_{f3} = I_S (i_{f2} - i_{r2})$$

BUT $V_{S3} = V_{D2} \Rightarrow i_{f3} = i_{r2}$

↑ FROM (1), (2)

b) $\therefore$ M2 (+1) IS IN TRIODE SINCE i_{r2} IS SIGNIFICANT.

$$I_{OUT} = I_S \, i_{f3} = I_S \, i_{r2} = I_S (i_{f2} - i_{r2})$$

$$2 I_S \, i_{r2} = I_S \, i_{f2}$$

$$i_{r2} = \frac{i_{f2}}{2} \quad (+1)$$

$$i_{f2} = i_{f1} = 0.089$$

$$i_{r2} = \frac{0.089}{2} \simeq 0.045 \quad (+1)$$

$$I_{OUT} = \frac{I_{in}}{2}$$