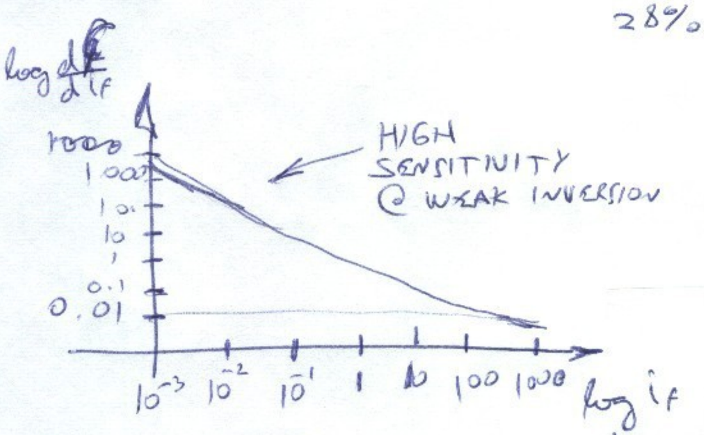
$$\Delta i_f = \frac{\Delta V_P}{V_T\left(\frac{\partial F}{\partial i_f}\right)} = \left[\frac{0.0036}{V_T}, \frac{0.099}{V_T}\right]$$

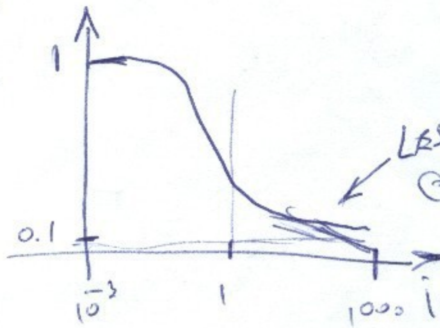
$$\Delta I_D = I_S \Delta i_f \simeq \left[0.14\mu A,\ 3.81\mu A\right]$$

28% 7.6% ← LESS EFFECT NEAR STRONG INVERSION

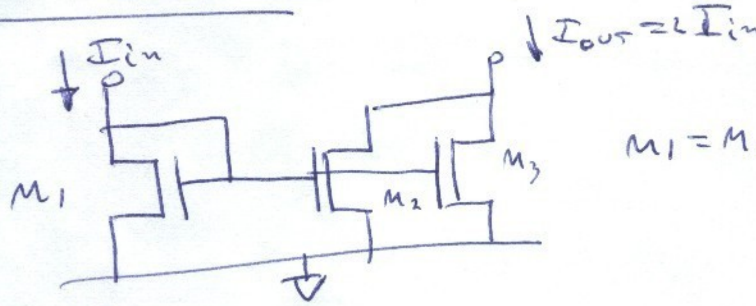


∴ MATCHING MORE DIFFICULT IN WEAK INVERSION.

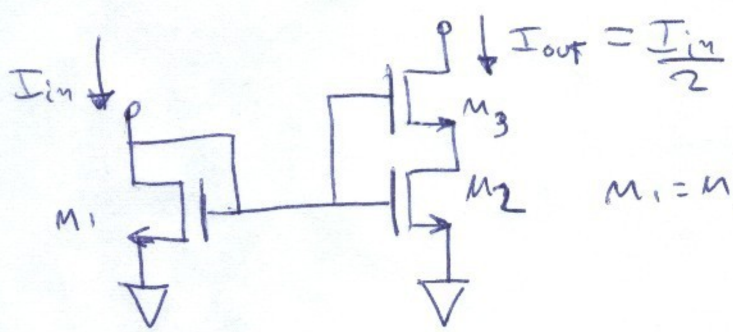
$$\frac{\Delta i_f}{i_f} = \frac{\Delta V_P}{V_T\, i_f\left(\frac{\partial F}{\partial i_f}\right)} \propto \frac{1}{i_f \frac{\partial F}{\partial i_f}}$$



CURRENT MIRROR GAIN



$M_1 = M_2 = M_3$



$M_1 = M_2 = M_3$

+

V_{OUT}

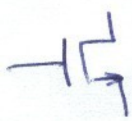
−

- $M_1 = M_2$ AND $V_{G1} = V_{G2} \Rightarrow i_{f1} = i_{f2}$
- $i_{r_2} = i_{f_3}$ SINCE $V_{D_2} = V_{S_3}$ AND $M_2 = M_3$ (M_2 TRIODE)
- ASSUMING V_{OUT} IS HIGH ENOUGH, $i_{r_3} \approx 0$ (M_3 ACTIVE REGION)

$$\therefore i_{f_3} = i_{f_2} - i_{r_2} = i_{f_2} - i_{f_3}$$

$$\left.\therefore i_{f_3} = \frac{i_{f_2}}{2} = \frac{i_{f_1}}{2}\right] \Rightarrow I_{OUT} = \frac{I_{in}}{2}$$

- THE SAME WORKS FOR ANY NUMBER OF TRANSISTORS IN SERIES.

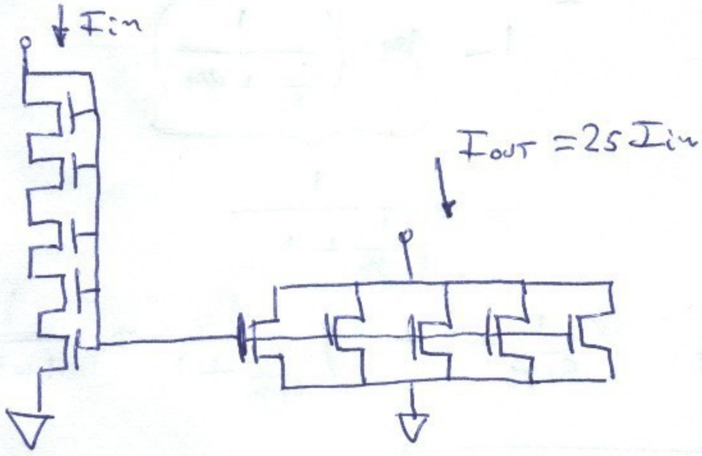


$\longrightarrow$ N TRANSISTORS IN // EQUIVALENT TO $N\frac{W}{L}$

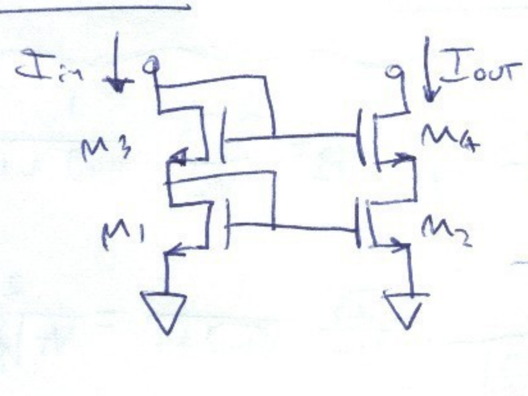
N TRANSISTORS IN SERIES EQ. TO $\frac{W}{NL}$

EXAMPLE: PROBLEM 5.4

IMPORTANT: TRANSISTORS ARE ALL EQUAL $\rightarrow$ GOOD MATCHING.



CASCODE CURRENT MIRRORS: TO GET HIGH OUTPUT RESISTANCE



V_O, M_1 AND M_2 OPERATE ALMOST @ EXACT SAME OPERATING POINT

$\therefore$ SAME DRAIN CURRENTS

. ASSUME ALL TRANSISTORS ACTIVE: $i_{r_{1,2,3,4}} \ll i_{f_{1,2,3,4}}$

. FROM TOPOLOGY IT FOLLOWS $i_{f_1} = i_{f_2} = i_{f_3} = i_{f_4} = i_f$ → SOLVE FOR V_{G1}

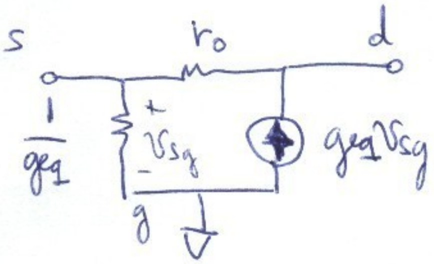
$$V_{DS4} > V_{DSAT} = V_T\left(\sqrt{1+i_f}+3\right)$$

$$\therefore V_{O_{MIN}} = V_{G1} + V_T\left(\sqrt{1+i_f}+3\right)$$

. OUTPUT RESISTANCE → USE SMALL-SIGNAL MODEL. (PAGE 14)

$g_m = \frac{g_{ms}}{n}$

$g_{mb} = g_{ms}\left(\frac{n-1}{n}\right)$



$g_{eq} = g_m + g_{mb}$

$$g_{eq} = g_{ms}\underbrace{\left(\frac{1}{n}+\frac{n-1}{n}\right)}_{=1}$$

$$= g_{ms}$$

$$R_o \cong r_{o2}\left(g_{ms4} r_{o4}\right) \rightarrow r_{o2} \text{ AMPLIFIED BY } g_{ms4} r_{o4}$$

$$r_o = \frac{L}{I_D\left|\frac{\partial X_D}{\partial V_{DS}}\right|} \simeq \frac{L}{I_S i_f\left|\frac{\partial X_D}{\partial V_{DS}}\right|}$$

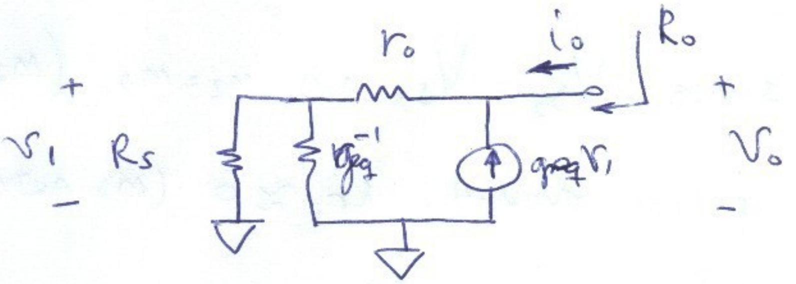
$$g_{ms} = \frac{2I_S}{V_T}\left(\sqrt{1+i_f}-1\right)$$

$$R_o \simeq \frac{L}{\cancel{I_S} i_f\left|\frac{\partial X_D}{\partial V_{DS}}\right|} \cdot \frac{2\cancel{I_S}}{V_T}\left(\sqrt{1+i_f}-1\right)\frac{L}{I_S i_f\left|\frac{\partial X_D}{\partial V_{DS}}\right|}$$

$$R_o \simeq \frac{2L^2}{I_S\left|\frac{\partial X_D}{\partial V_{DS}}\right|^2} \cdot \frac{\left(\sqrt{1+i_f}-1\right)}{i_f^2}$$ ← CAN GET A BETTER EXPRESSION

(EXAMPLE 5.5)

(11')



$$v_1 = V_o \frac{(R_s \| g_{eq}^{-1})}{r_o + (R_s \| g_{eq}^{-1})}$$

$$i_o = \frac{V_o}{r_o + (R_s \| g_{eq}^{-1})}\left[1 - g_{eq}(R_s \| g_{eq}^{-1})\right]$$

$$R_o = \frac{V_o}{i_o} = \frac{r_o + (R_s \| g_{eq}^{-1})}{1 - g_{eq}(R_s \| g_{eq}^{-1})}$$

$$r_o \gg g_{eq}^{-1} \Rightarrow R_o \simeq \frac{r_o}{1 - g_{eq}\underbrace{\left(\frac{1}{\frac{1}{R_s} + g_{eq}}\right)}_{= \frac{1}{\frac{1}{R_s g_{eq}} + 1}}}$$

DENOMINATOR: $\left(\frac{1}{R_s g_{eq}} \cancel{+1} \cancel{-1}\right)\frac{1}{\frac{1}{R_s g_{eq}} + 1} = \frac{1}{1 + R_s g_{eq}} \simeq \frac{1}{R_s g_{eq}}$

$$\boxed{\therefore\ R_o \simeq (r_o g_{eq}) R_s}$$

$$r_o = \frac{L}{I_D \left|\frac{\partial X_D}{\partial V_{DS}}\right|}, \quad g_{eq} = \frac{2 I_D}{V_T} \frac{1}{\sqrt{1+i_f} + 1}$$

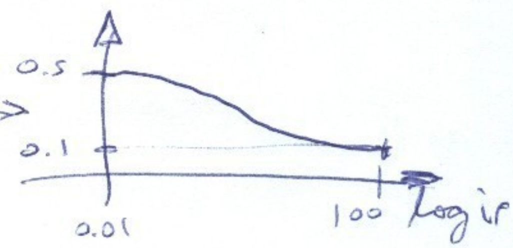
$$\frac{r_o}{g_{eq}^{-1}} = r_o g_{eq} \gg 1 \rightarrow \frac{L}{\cancel{I_D} \left|\frac{\partial X_D}{\partial V_{DS}}\right|} \cdot \frac{2 \cancel{I_D}}{V_T} \frac{1}{\sqrt{1+i_f} + 1} = \frac{2L}{V_T \left|\frac{\partial X_D}{\partial V_{DS}}\right| (\sqrt{1+i_f} + 1)} > 1$$

$$g_{ms} = \frac{2I_D}{V_T} \frac{1}{\sqrt{1+i_f}+1} \quad \text{(FOR SATURATION ONLY)}$$

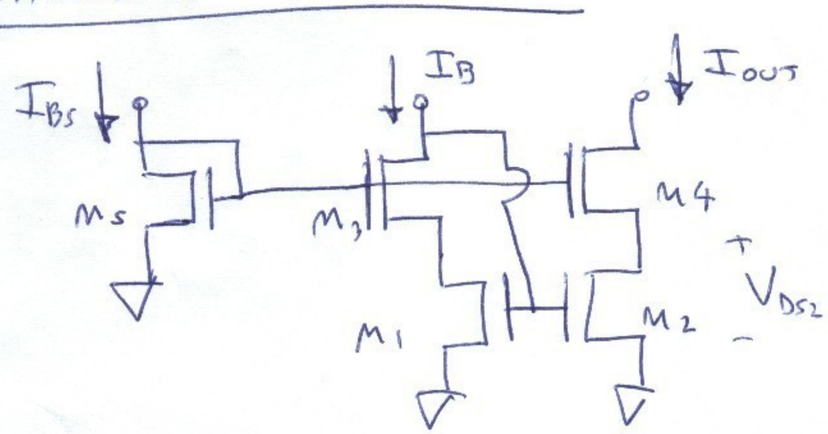
$$R_o \simeq \frac{2\cancel{I_D}}{V_T} \frac{1}{\sqrt{1+i_f}+1} \cdot \frac{L^2}{I_D^{\cancel{2}} \left|\frac{\partial x_D}{\partial V_{DS}}\right|^2}$$

- ASSUME I_D = CONSTANT AND WE ONLY CAN CHANGE INVERSION LEVEL

$$R_o \simeq \frac{2L^2}{V_T I_D \left|\frac{\partial x_D}{\partial V_{DS}}\right|^2} \cdot \frac{1}{\sqrt{1+i_f}+1} \longrightarrow$$



HIGH-SWING CASCODE MIRROR



$$V_{P5} = V_{P3}$$

$$V_{P5} = V_T\left[\sqrt{1+i_{f5}} - 2 + \ln\left(\sqrt{1+i_{f5}} - 1\right)\right]$$

$$= V_{P4} = V_{S4} + \cancel{V_T}\, \mathcal{G}(i_{f4})$$

$$V_{S4} = V_{DS_{SAT2}} + \Delta V \implies \text{CHOOSE } \Delta V \simeq V_T$$

$$V_{P5} = V_{DS_{SAT2}} + V_T\left(1 + \mathcal{G}(i_{f4})\right)$$

BUT $V_{DS_{SAT2}} = V_T\left(\sqrt{1+i_{f2}} + 3\right)$

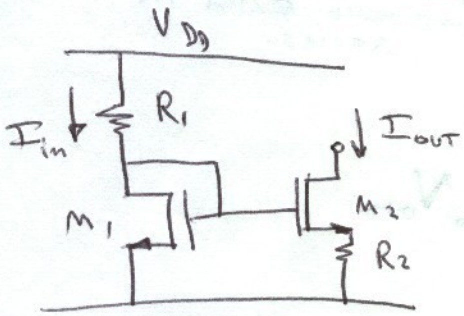
$$V_{P5} = V_T\left[\sqrt{1+i_{f2}} + 3 + 1 + \mathcal{G}(i_{f4})\right] \longrightarrow \text{IF } M4 \equiv M_2 \text{ THEN } i_{f4} = i_{f2}$$

GIVEN $i_{f2}, i_{f4} \implies$ SOLVE FOR i_{f5}

VOLTAGE CURRENTS AND REFERENCES

LOW-CURRENT BIASING

- WIDLAR CURRENT SOURCE. → LOW OUTPUT CURRENT WITH NOT SO LARGE RESISTOR.



$$V_{GS_1} - V_{GS_2} - I_{OUT} R_2 = 0$$

$$V_{GS} = V_{OV} + V_T$$

$$\therefore I_{OUT} R_2 + V_{OV_2} - V_{OV_1} = 0$$

$$\sqrt{I_{OUT}} = \frac{-\sqrt{\frac{2}{k'(\frac{W}{L})_2}} + \sqrt{\frac{2}{k'(\frac{W}{L})_2} + 4R_2 V_{OV_1}}}{2R_2}$$

EXAMPLE: $I_{in} = 100\mu A$, $R_2 = 4k\Omega$, $k' = 200 \frac{\mu A}{V^2}$, $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 25$

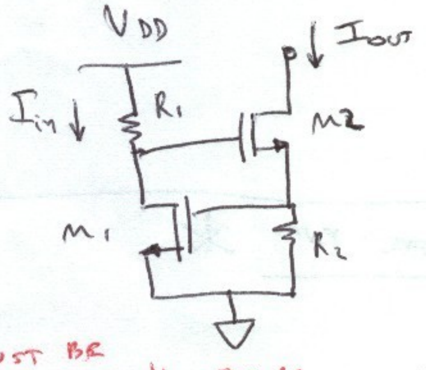
$$\sqrt{I_{OUT}} = 5\sqrt{\mu A} \Rightarrow I_{OUT} = 25\mu A$$

$$V_{OV_2} = 0.1V > 2nV_T = 78mV$$

(STRONG INVERSION)
(NOT REALLY)

- CURRENT SOURCE USING A $\underbrace{V_{th}}_{\text{THRESHOLD}}$ STANDARD

EXAMPLE (*) PAGE 36"



$$I_{OUT} = \frac{V_{GS_1}}{R_2} = \frac{V_t + \overbrace{\sqrt{\frac{2I_{in}}{k'(\frac{W}{L})_1}}}^{V_{OV_1}}}{R_2}$$

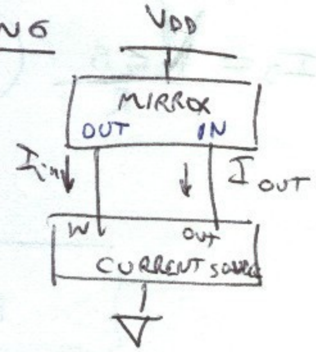
IF $V_{OV_1} << V_t$ $\Rightarrow$ OUTPUT CURRENT ALMOST INDEPENDENT OF I_{in}.

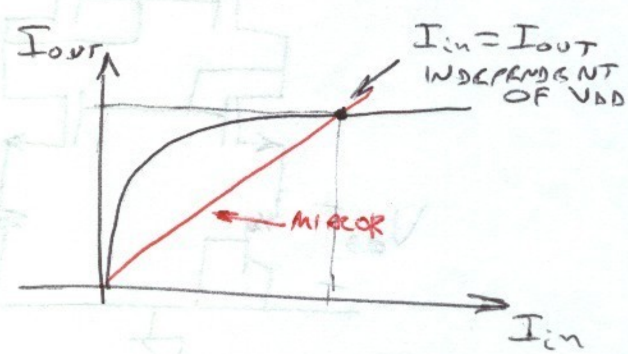
→ CANNOT BE ACHIEVED WITH STRONG INVERSION.

$\left(\frac{W}{L}\right)_1$ MUST BE LARGER FOR V_{OV_1} TO BE SMALL.

$\therefore$ LESS DEPENDENCE ON VDD

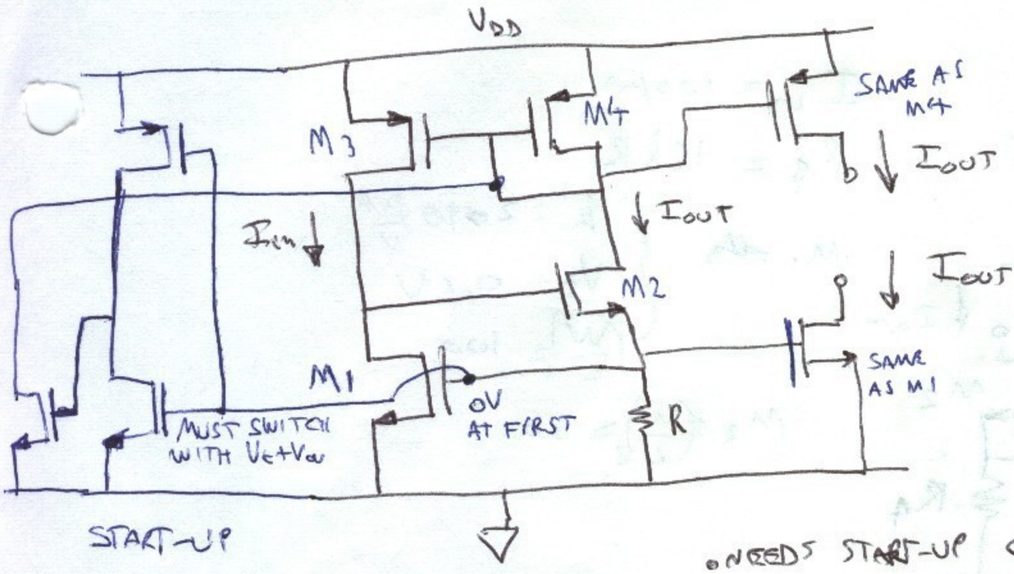
SELF-BIASING





- SELF-BIASING V_t REFERENCE

$M_3 = M_4$



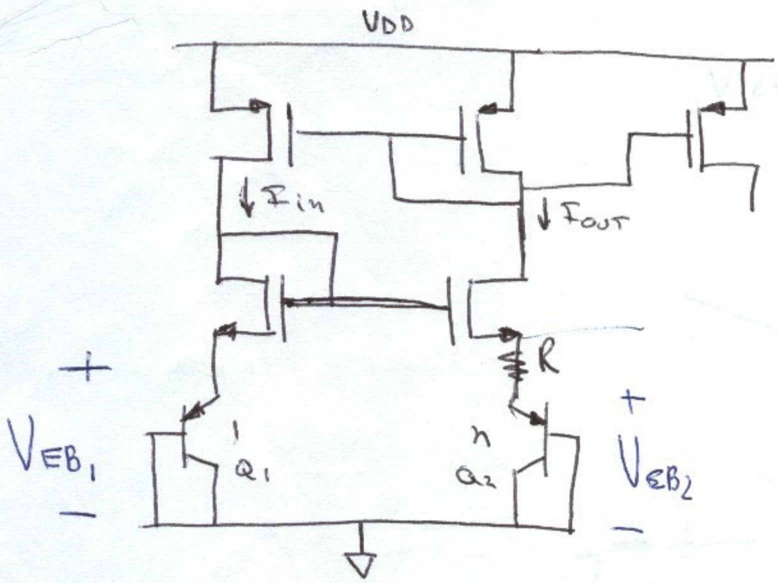
(NPTAT)

DECREASES WITH TEMP. SINCE V_{th} DECREASES

$$I_{OUT} = \frac{V_{th} + \sqrt{\dfrac{2 I_{OUT}}{k'\left(\frac{W}{L}\right)_1}}}{R}$$

- NEEDS START-UP CIRCUIT (PAGE 312)

- SELF-BIASING USING V_T



$$I_{OUT} = \frac{V_T \ln(n)}{R}$$

$$\cancel{I_S} \exp\left(\frac{V_{BE1}}{V_T}\right) = n \cancel{I_S} \exp\left(\frac{V_{BE2}}{V_T}\right)$$

$$\frac{V_{BE1}}{V_T} = \ln(n) + \frac{V_{BE2}}{V_T}$$

BUT $V_{BE_1} = V_{BE_2} + I_{OUT} R$

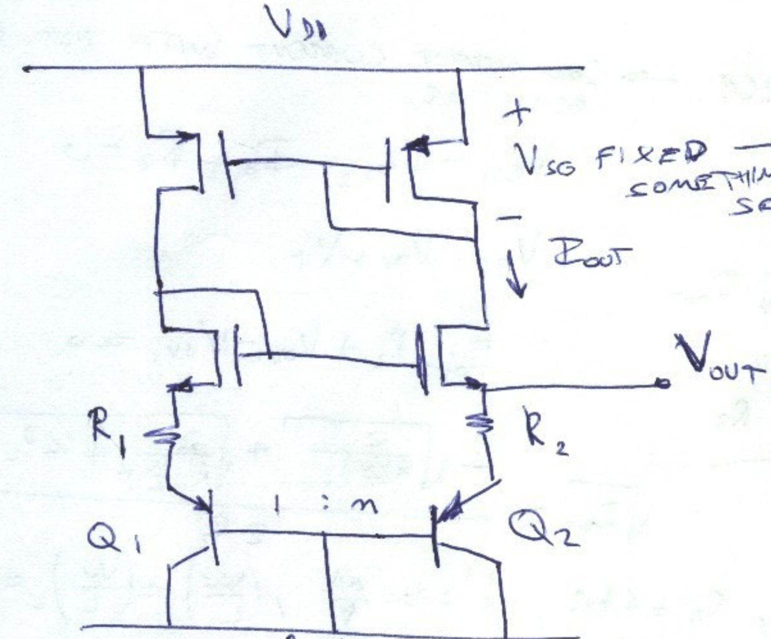
$$\cancel{V_{BE_2}} + I_{OUT} R = V_T \ln(n) + \cancel{V_{BE_2}}$$

$$\boxed{I_{OUT} = \frac{V_T \ln(n)}{R}}$$

(INCREASES WITH TEMP.)

(PTAT)

BANDGAP REFERENCE



V_{SG} FIXED → NEED SOMETHING EXTRA IN SERIES.

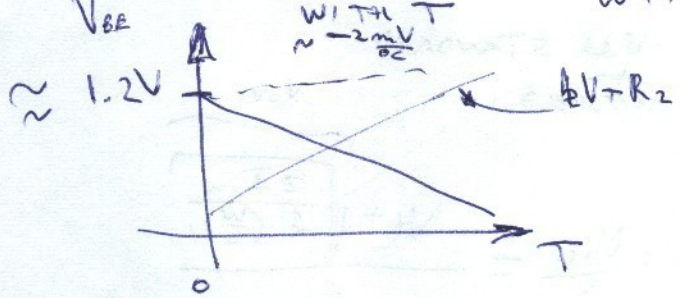
$$I_{OUT} = \frac{V_T \ln(n)}{(R_2 - R_1)} = k\, V_T$$

$$V_{OUT} = \underbrace{V_{BE_2}}_{\text{DECREASES WITH T}} + \underbrace{k\, V_T\, R_2}_{\text{INCREASES WITH T}} \cong \overbrace{1.2\,V}^{\text{BANDGAP OF SILICON}}$$

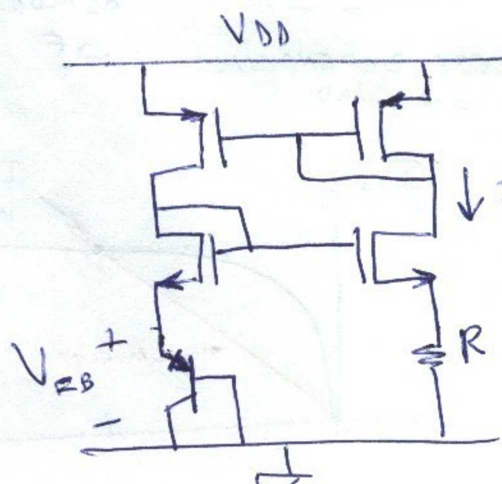
EXTRAPOLATED @0K

INCREASES WITH T → MAKE $+2\frac{mV}{°C}$

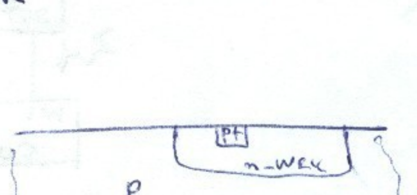
DECREASES WITH T $\sim -2\frac{mV}{°C}$



SELF-BIASED C. SOURCE USING PARASITIC ~~LATERAL~~ PNP *



$$\downarrow I_D = \frac{V_{EB}}{R} \quad (\text{NPTAT})$$



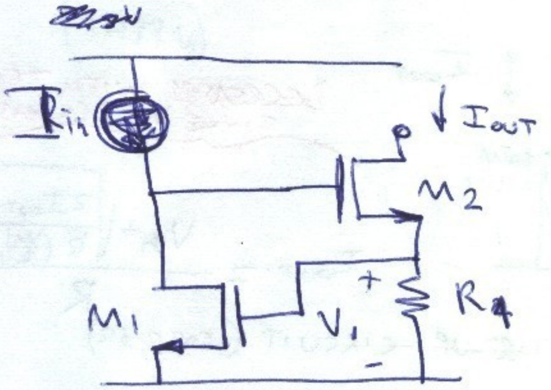
EXAMPLE

$I_{in} = 100\mu A$

$R_q = 16k\Omega$

M_1 M_2: $k' = 210 \frac{\mu A}{V^2}$, $V_t = 0.6V$, $\left(\frac{W}{L}\right)_1 = 100$

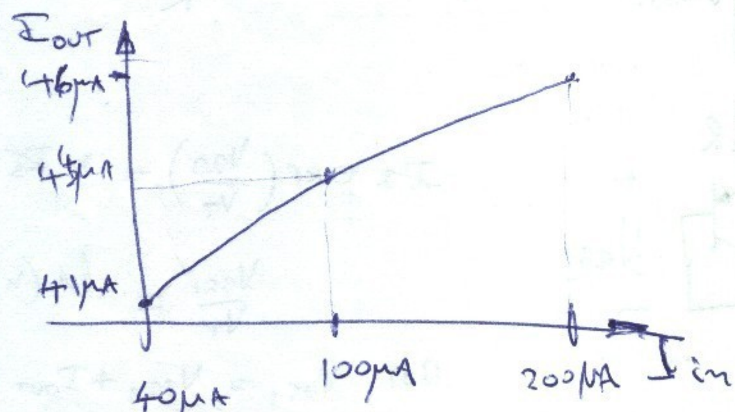
$M_2 : \left(\frac{W}{L}\right)_2 = 30$

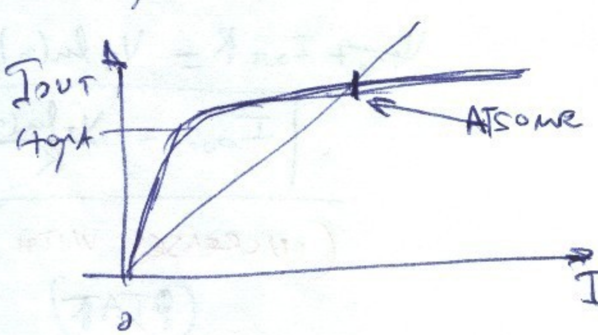


$$V_{ov_1} = \sqrt{\frac{2 I_{in}}{k'\left(\frac{W}{L}\right)_1}} \cong 97.6\,mV$$

$$V_1 = V_{tn} + V_{ov_1} \cong 0.698\,V$$

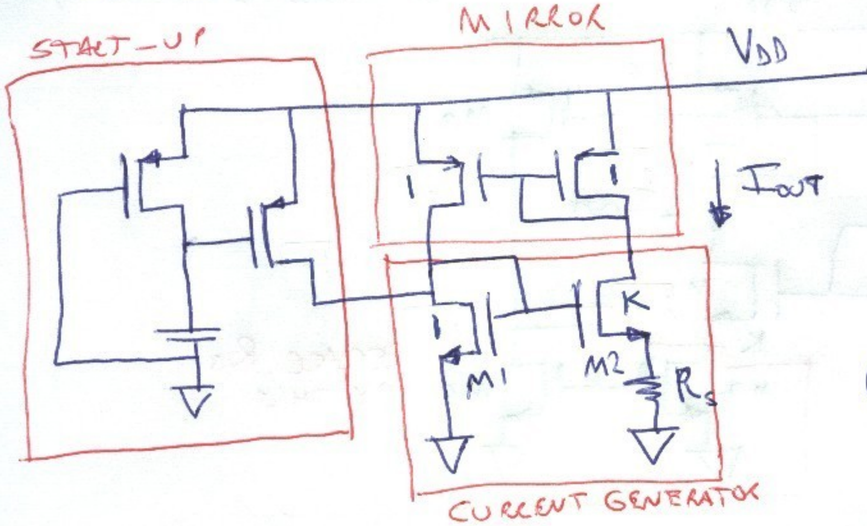
$$\therefore I_{OUT} \cong 43.6\,\mu A$$





AT SOME POINT $I_{in} = I_{OUT}$

ANOTHER SELF-BIASED CURRENT SOURCE (PTAT)



$K > 1$

(Vittoz et al.)

M1, M2 WORK IN SATURATION, $I_{D1} = I_{D2}$ DUE TO MIRROR

$$I_{D1} = I_{D2} = I_{OUT}$$

$$i_{f1} = \frac{I_{OUT}}{I_S}, \quad i_{f2} = \frac{I_{OUT}}{K I_S}$$

$$\frac{V_{P1}}{V_T} = \mathcal{G}\left(\frac{I_{OUT}}{I_S}\right), \quad \frac{V_{P1} - I_{OUT} R_S}{V_T} = \mathcal{G}\left(\frac{I_{OUT}}{K I_S}\right)$$

$$\therefore \frac{I_{OUT} R_S}{V_T} = \mathcal{G}\left(\frac{I_{OUT}}{I_S}\right) - \mathcal{G}\left(\frac{I_{OUT}}{K I_S}\right)$$

FOR $i_{f1} \ll 1$ (DEEP SUBTHRESHOLD):

$$\boxed{I_{OUT} \cong V_T \frac{\ln K}{R_S}}$$

(PTAT) (ASSUMING R_S DOES NOT CHANGE MUCH WITH TEMP.)

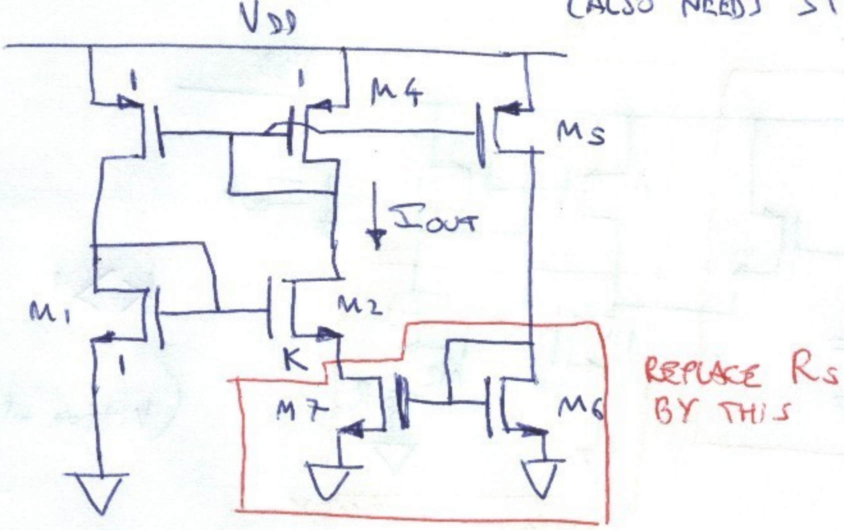
FOR $i_{f1} \gg 10$ (STRONG INVERSION)

$$I_{OUT} \cong \frac{V_T^2}{R_S^2 I_{S1}} \left(1 - \sqrt{\frac{1}{K}}\right)^2$$

(ASSUMING R_S CONSTANT, I_{OUT} INCREASES WITH TEMP. BUT NOT LINEARLY.)

OGUEY'S CURRENT SOURCE

(ALSO NEEDS START-UP)



ADVANTAGE: NO RESISTOR

- ASSUME M1, M2 IN DEEP SUBTHRESHOLD:

$$V_{S2} \cong V_T \ln K$$

- M7 WORKS IN TRIODE REGION
- IF M7, M6 OPERATE IN STRONG INVERSION,

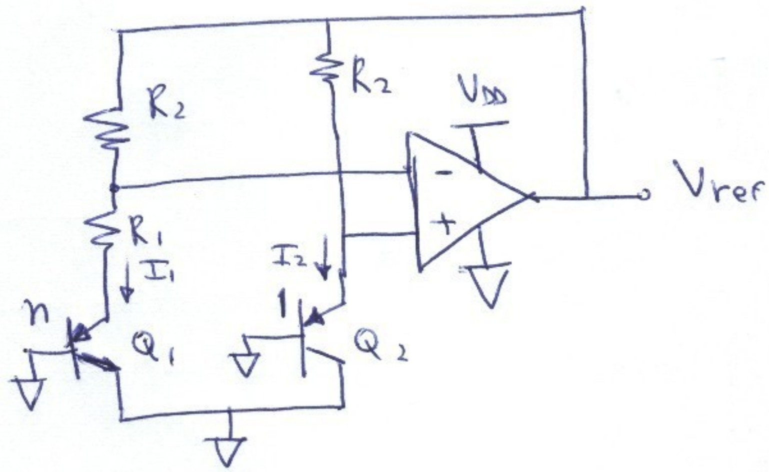
$$I_{OUT} \simeq \mu_n C_{ox} n \frac{V_T^2}{2} \ln^2 K \times C_1$$

CONSTANT THAT DEPENDS ON TRANSISTOR RATIOS (M4, 5, 6, 7)

$I_{OUT} \propto I_{SQ} \Rightarrow$ CONSTANT INVERSION LEVEL WITH TEMPERATURE.

$$\boxed{I_{OUT} = I_{SQ} \ln^2 K \times C_1}$$

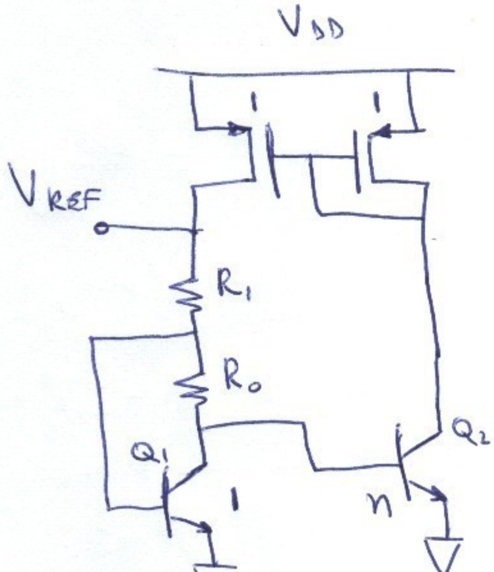
ALTERNATIVE BANDGAP REFERENCES



$I_1 = I_2$ SINCE $V^+ = V^-$

$$V_{REF} = V_{EB2} + \frac{R_2}{R_1} V_T \ln n$$

(WORKS WITH LOWER SUPPLY VOLTAGE)



$$I_{C1} \simeq \frac{V_T}{R_0} \ln K$$

$$V_{Ref} \simeq V_{BE,1} + \frac{R_1}{R_0} V_T \ln K$$

(NEGLECTING BASE CURRENTS)

• NEEDS NPN TRANSISTORS