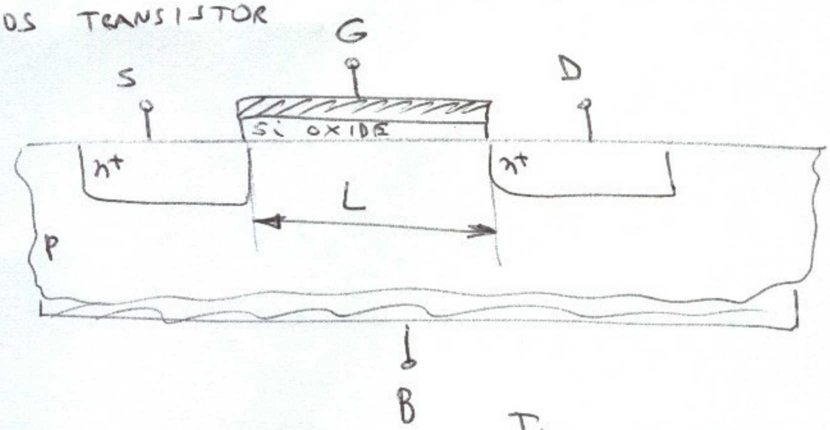
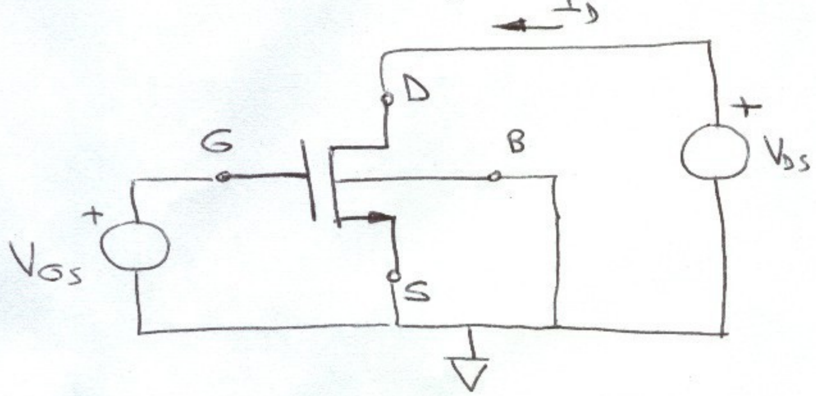
# ENGI 5131 : BASIC MOSFET MODEL

NMOS TRANSISTOR




N-CHANNEL TRANSISTOR




$V_t$ : THRESHOLD VOLTAGE

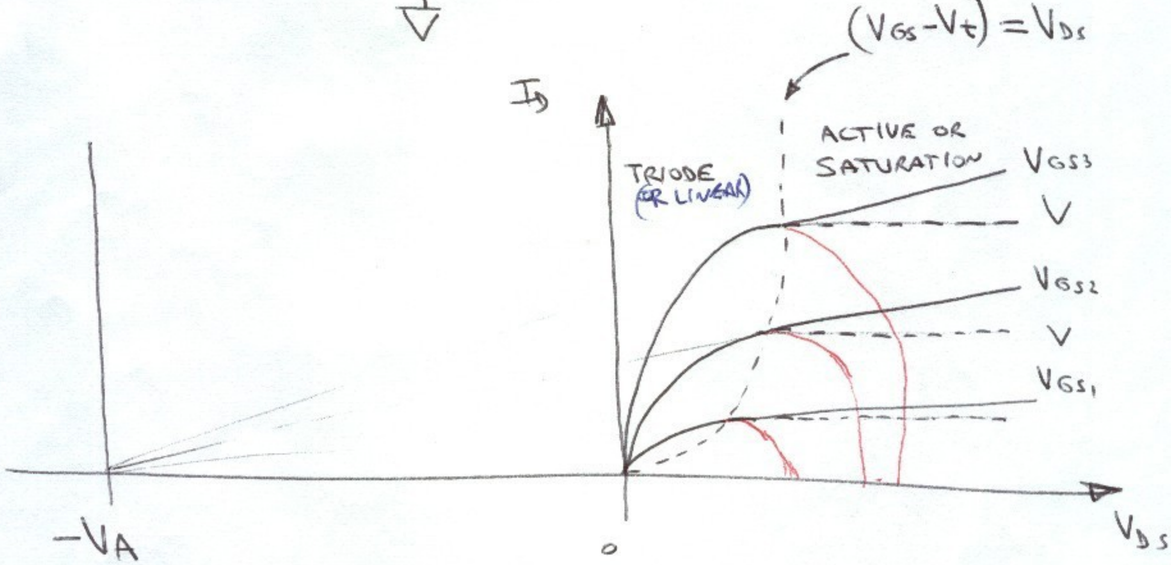



FIG. (1)

- TRIODE REGION

$$I_D = K\left[2(V_{GS}-V_t)V_{DS} - V_{DS}^2\right], \quad (V_{GS}-V_t) \geq V_{DS}$$

- SATURATION REGION

$$I_D \cong K(V_{GS}-V_t)^2$$

↑ NEGLECTING OUTPUT RESISTANCE

↑ SAME AS THIS WITH $V_{DS} = V_{GS} - V_t$

$$I_D = k\left(V_{GS}-V_t\right)^2\left(1+\lambda V_{DS}\right)$$

$\lambda$: CHANNEL-LENGTH MODULATION FACTOR $\left(\frac{1}{V}\right)$

$\lambda = \frac{1}{V_A}$ , $V_A$ : EARLY VOLTAGE. → LATER WE WILL SEE HOW TO CALCULATE THIS.

(CUT OFF: $V_{GS} < V_t$)

OVERDRIVE VOLTAGE : $\boxed{V_{ov} := V_{GS} - V_t}$ → BODY EFFECT CONSIDERED LATER.

$$K = \frac{\mu_n C_{ox}}{2}\frac{W}{L}, \quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$k' := \mu_n C_{ox}$ → CONSTANT FOR A GIVEN FAB. PROCESS.

$\epsilon_{ox} \simeq 3.45 \times 10^{-11} \frac{F}{m}$

~~tox~~ EXAMPLE

$t_{ox} = 100\,\text{Å}$ $\left(1\,\text{Å} = 10^{-10}\,m\right)$ , $\mu_n = 450 \frac{cm^2}{Vs} = 0.045 \frac{m^2}{Vs}$

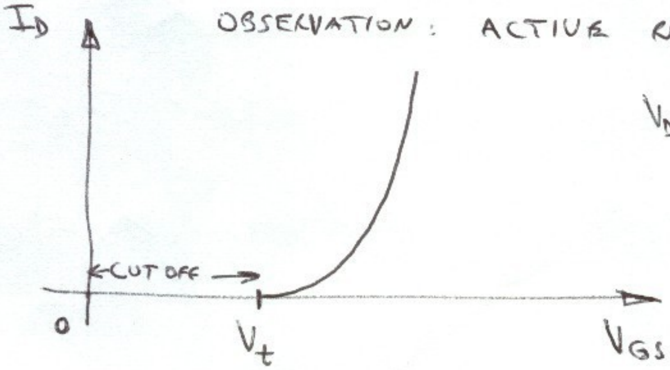
↳ $C_{ox} = 3.45 \frac{fF}{(\mu m)^2}$ $\left(1\,fF = 10^{-15}\,F\right)$

$\boxed{k' = 155 \frac{\mu A}{V^2}}$ $V_t = 0.6\,V$

NOTE $\left(V_{GS} - V_t\right) = V_{DS}$ IN FIG.1 IS A PARABOLA:

$$V_{GS} - V_t = V_{ov} = V_{DS} \Rightarrow \boxed{I_D = \frac{k'}{2}\frac{W}{L}V_{ov}^2 = \frac{k'}{2}\frac{W}{L}V_{DS}^2}$$

OBSERVATION: ACTIVE REGION IS WHEN $V_{DS} \geq V_{ov}$




$V_{DS}$ = CONSTANT.

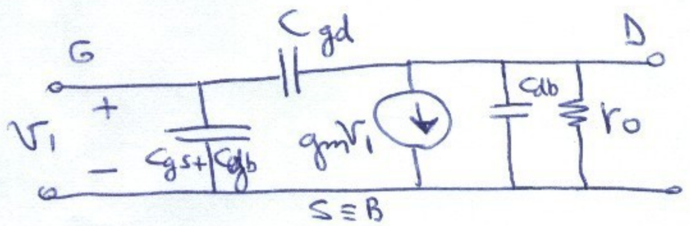
## MORE ON OUTPUT RESISTANCE

- THE EFFECTIVE CHANNEL LENGTH DEPENDS ON $V_{DS}$: AS $V_{DS}$ INCREASES, THE DEPLETION REGION AT THE DRAIN INCREASES. ∴ EFFECTIVE LENGTH ($L_{eff}$) DECREASES.

$X_d$: LENGTH OF DEPLETION REGION

$$V_A \cong L \left(\frac{\partial X_d}{dV_{DS}}\right)^{-1} \qquad \therefore V_A \propto L$$

## SMALL-SIGNAL MODEL FOR $V_{SB}=0$




ACTIVE REGION:

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}(V_{OV}) = \sqrt{2 I_D \mu C_{ox}\frac{W}{L}}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{L}{I_D\left(\frac{\partial X_d}{\partial V_{DS}}\right)} \qquad \left(=\left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1}\right)$$

$$C_{gs} \cong \frac{2}{3} W L C_{ox} + C_{OV}$$ ($C_{OV}$: OVERLAP CAPACITANCE)

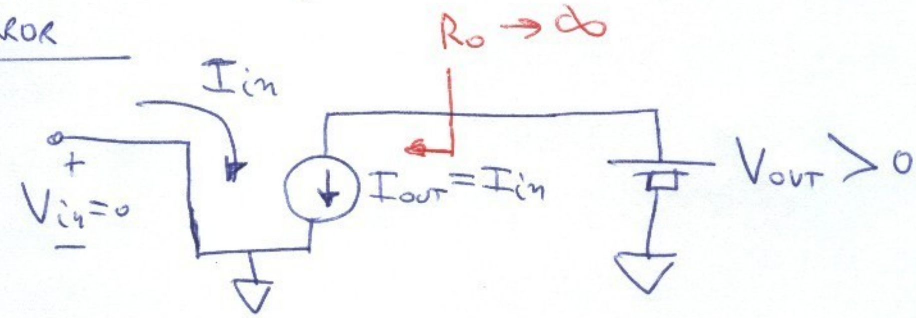
$$C_{gd} = C_{OV}$$

$C_{db}$: DRAIN DEPLETION CAPACITANCE (JUNCTION)

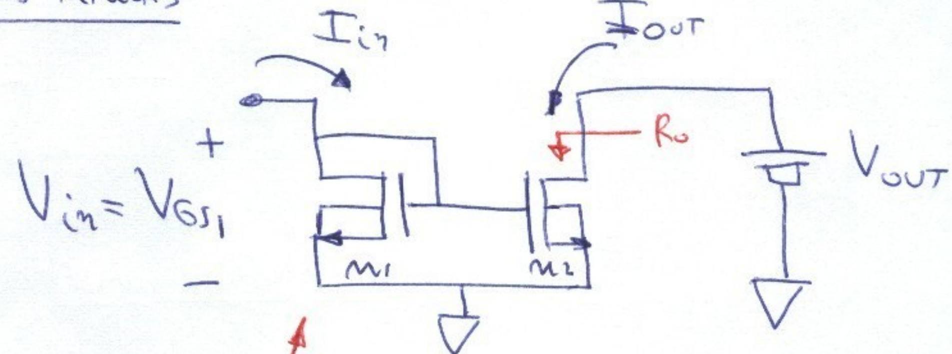
$C_{gb}$: POLY – BULK CAPACITANCE OUTSIDE OF CHANNEL AREA

## SIMPLE CURRENT MIRROR

IDEALLY:




## SIMPLE MOS MIRRORS



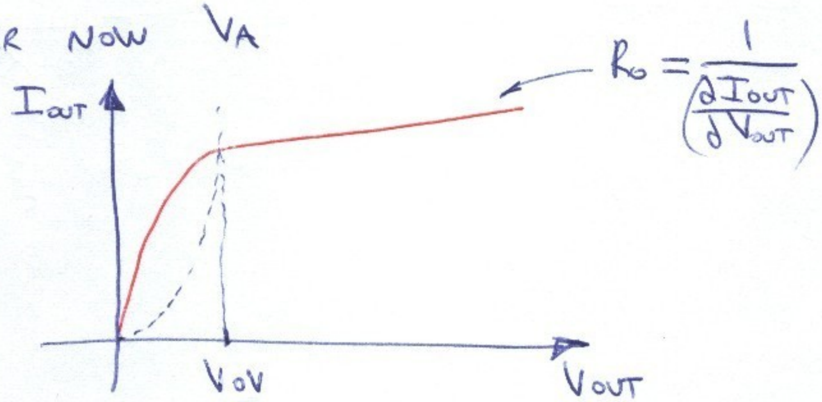

EXPLAIN DIODE-CONNECTED → EXERCISE PAGE 6

- ASSUME $M_1 = M_2$ AND $V_A$ VERY LARGE ($\lambda \simeq 0$)

  THEN $I_{OUT} \approx I_{in}$ IF $V_{OUT} \geq V_{OV_2}$ (SINCE $V_{GS1} = V_{GS2}$)

$$I_{in} = \frac{k'}{2}\frac{W}{L}V_{OV}^2 \Rightarrow V_{OV} = \sqrt{\frac{2 I_{in}}{k'\left(\frac{W}{L}\right)}} \quad (\leq V_{OUT})$$

- CONSIDER NOW $V_A$




$$I_{OUT} = \frac{k'}{2}\frac{W}{L}V_{OV}^2\left(1 + \frac{V_{OUT}}{V_A}\right)$$

$$\frac{\partial I_{OUT}}{\partial V_{OUT}} = \frac{k'}{2}\frac{W}{L}V_{OV}^2 \frac{1}{V_A} \simeq \frac{I_{OUT}}{V_A} \Rightarrow R_o = \frac{V_A}{I_{OUT}} = r_{o2}$$

$$R_o = r_{o2} = \frac{L}{\left(\frac{\partial X_d}{\partial V_{DS}}\right) I_{OUT}}$$

- SUMMARY:

1. $V_{in}$ NOT ZERO $\rightarrow \left[\overbrace{\sqrt{\frac{2 I_{in}}{k' \frac{W}{L}}}}^{V_{ov}} + V_t\right]$
   - CAN REDUCE BY INCREASING $\frac{W}{L}$ BUT $V_{OV} > 100mV$ TO KEEP EQUATIONS VALID (STRONG INVERSION)

2. $I_{OUT} \neq I_{in}$ DUE TO $R_o$ NOT $\infty$.
   - DIFFERENCE IS $\Delta I = \frac{(V_{OUT} - V_{in})}{R_o}$

$$= \frac{(V_{OUT} - V_{OV} - V_t)}{L} \times \left|\frac{\partial X_d}{\partial V_{DS}}\right| \times I_{OUT}$$

   - CAN BE REDUCED BY INCREASING L

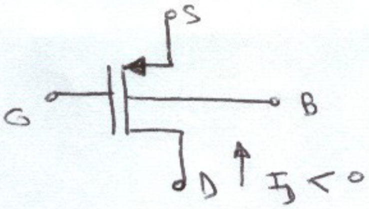
3. $I_{OUT} \neq I_{in}$ DUE TO MISSMATCH IN $M_1, M_2$

4. MINIMUM $V_{OUT}$ NOT ZERO $\rightarrow V_{OV}$ (SAME CONSIDERATIONS AS $V_{in}$)

(JUMP TO PAGE 29) → EXAMPLE P. 4.3

- SMALL-SIGNAL → JUMP TO PROBLEM 3.4 PAGE 15

## PMOS TRANSISTOR




$V_t < 0$

SAME EQUATIONS THAN FOR NMOS WORK IF SIGNS OF VOLTAGES AND CURRENTS ARE INVERTED.

$$I_D = -\frac{k'}{2}\frac{W}{L}\left(V_{GS} - V_t\right)^2$$

$V_{DS} < 0$

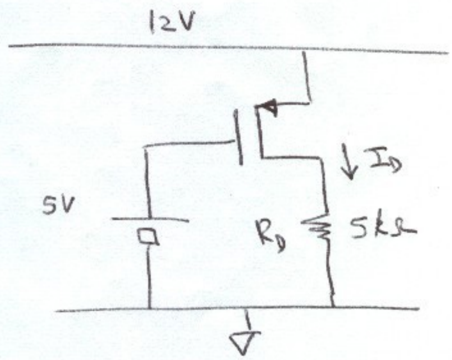
$V_{GS} < 0$

$k'_p < k'_n$ IN GENERAL FOR A GIVEN TECHNOLOGY. THIS IS BECAUSE MOST PROCESSES ARE OPTIMIZED FOR NMOS TRANSISTORS

↓

DUE TO $\mu_p < \mu_n$ (EVERYTHING ELSE IS THE SAME)

**EXAMPLES**: 7.6, 7.7 AND EXERCISE (PAGES 4, 5, 6 2007) → SHOW THESE FIRST




$V_{t_0} = -1V$

$k' = 15\frac{mA}{V^2}$

$\frac{W}{L} = 5$

$V_{SG} = 12V - 5V = 7V \Rightarrow V_{OV} = -6V$

ASSUME ACTIVE REGION: $I_D = \frac{k'}{2}\frac{W}{L}\left(-7V + 1V\right)^2 = 1.35\,mA$

$V_{R_D} = 1.35\,mA \times 5k\Omega = 6.75V \Rightarrow V_{SD} = 12V - 6.75V = 5.25V < |V_{OV}|$

∴ TRANSISTOR NOT IN ACTIVE REGION → TRIODE.

$$\begin{cases} I_D = \frac{k'}{2}\frac{W}{L}\left[2\,|V_{OV}|\,V_{SD} - V_{SD}^2\right] \\ V_{SD} = 12V - I_D R_D \end{cases}$$

→ SOLVE 2 EQ. 2 UNKNOWNS.

$$C_{gs} = \frac{2 C_{ox} W L}{3}$$

$$r_o = \frac{V_A}{I_D} = \frac{1}{\lambda I_D}$$

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \gamma \sqrt{\frac{k'(W/L)\, I_D}{2(2\phi_f + V_{SB})}}$$ (SECOND-ORDER EFFECT) (BODY EFFECT)

$$\frac{g_{mb}}{g_m} = \chi = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}, \quad 0.1 \leq \chi \leq 0.3$$

(PROBLEM 1.15 FOR PRACTICE)

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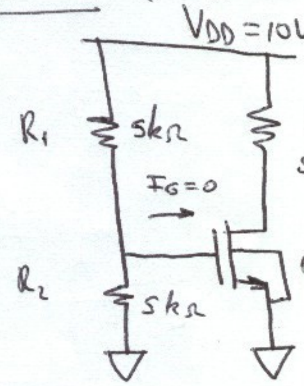
EXAMPLE 7.6 (SPENCER & GHAUSI)




FIND $I_D$

$M_1$: $k' = 2 \times 10^{-5} \frac{A}{V^2} = 20 \frac{\mu A}{V^2}$, $\frac{W}{L} = 5$, $V_{to} = 1V$ $\Rightarrow V_{ov} = 4V$

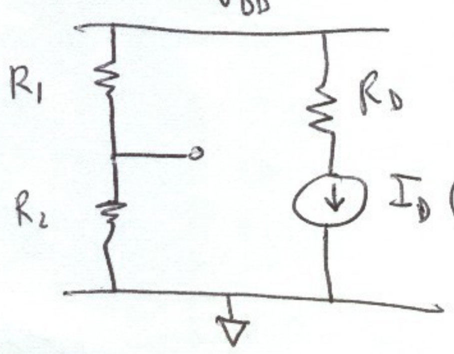
NOTE $V_{GS}$ MAY BE QUITE LARGE.

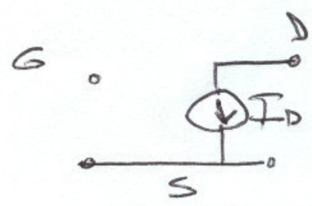
$$V_G = V_{DD} \frac{R_2}{R_1 + R_2} = 5V = V_{GS}$$

$$I_D = \frac{k'}{2} \frac{W}{L} (V_{GS} - V_t)^2 \quad \text{IF} \quad \underbrace{V_{GS} - V_t}_{V_{ov}} \leq V_{DS}$$

↑ TRY SINCE $V_{DS}$ IS UNKNOWN ANYWAY

$$I_D = 800\mu A \Rightarrow V_{DS} = 10V - 800\mu A \times 5k\Omega = 6V > V_{ov} \text{ (ok)}$$

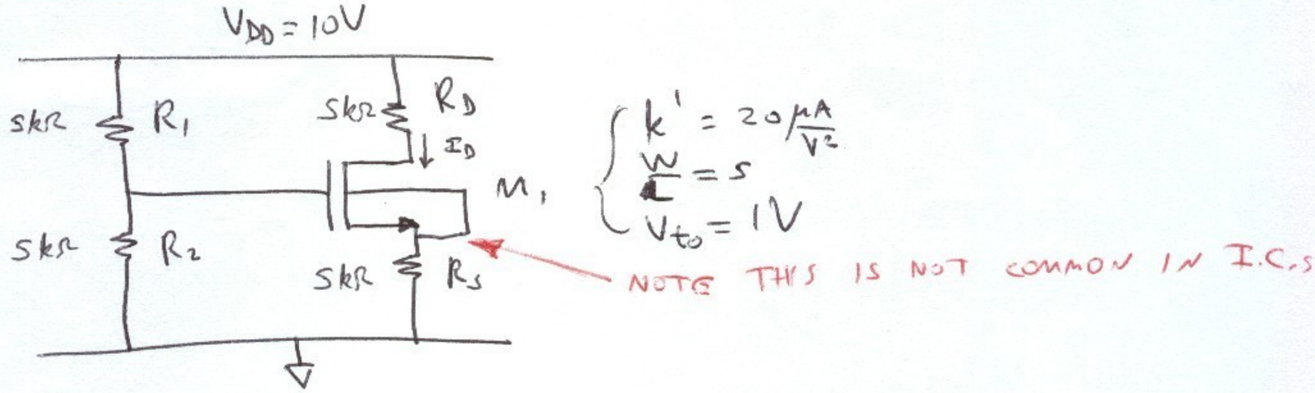


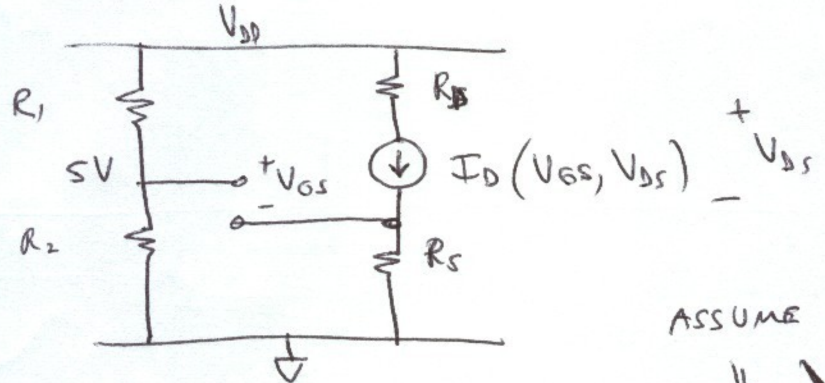

EQUIVALENT CIRCUIT (LARGE SIGNAL, L.F.)

EXAMPLE 7.7 (S&G.)






$$\begin{cases} k' = 20 \frac{\mu A}{V^2} \\ \frac{W}{L} = 5 \\ V_{t0} = 1V \end{cases}$$




ASSUME FOR NOW

$V_{DS} > V_{ov}$

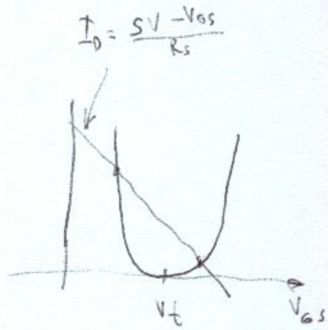
$$\therefore I_D = \frac{k'}{2}\frac{W}{L}(V_{ov})^2, \quad V_{ov} = \underbrace{\overset{V_G}{5V} - I_D R_s}_{=V_{GS}} - V_t$$

$$I_D = \frac{k'}{2}\frac{W}{L}(V_G - I_D R_s - V_t)^2$$

$$\frac{2}{k'}\frac{L}{W} I_D = (V_G - V_t)^2 - 2(V_G - V_t) I_D R_s + I_D^2 R_s^2$$

$$R_s^2 I_D^2 - \left[\frac{2}{k'}\frac{L}{W} + 2(V_G - V_t)R_s\right] I_D + (V_G - V_t)^2 = 0$$

$$25M\Omega^2 I_D^2 - 60k\frac{V^2}{A} I_D + 16V^2 = 0$$




$$I_{D_{1-2}} = \frac{60k \pm \sqrt{3.6\times10^9 - 1.6\times10^9}}{50M} = \begin{cases} 2.09\ mA \\ 306\ \mu A \end{cases}$$

$V_{R_D} = 5k\Omega\, I_D = 10.45V$ @ $I_D = 2.09mA$ (NOT POSSIBLE)

$\therefore I_D = 306\mu A \Rightarrow V_{DS} = 10V - 306\mu A \times 10k\Omega =$

$= 6.94V \rightarrow$ ACTIVE REGION

$V_{GS} = 3.47V \Rightarrow V_{ov} = 2.47V$

- QUESTION: WHAT IS THE MAXIMUM $R_D$ TO KEEP M1 IN ACTIVE REGION?

$$V_{DS} = V_{OV} = 2.47V \Rightarrow 2.47V = 10V - 306\mu A\,(R_D + 5k\Omega)$$

$$\boxed{R_D = 19.6\,k\Omega}$$

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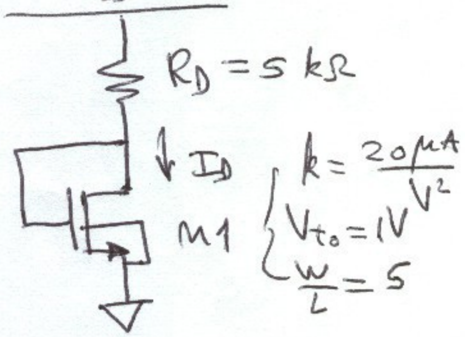
- EXERCISE:

CALCULATE $I_D$, $V_{DS}$ AND STATE REGION OF OPERATION:



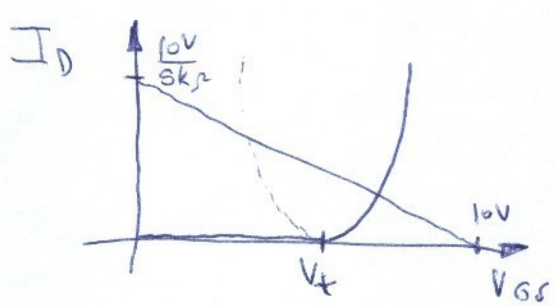

SOLUTION: $V_{GS} = V_{DS} \Rightarrow V_{DS} > V_{GS} - V_t = V_{OV} \rightarrow$ ACTIVE REGION (ALWAYS)

$$I_D = \frac{k'}{2}\frac{W}{L}\Big(\underbrace{V_{DD} - I_D R_D}_{= V_{GS}} - V_t\Big)^2$$

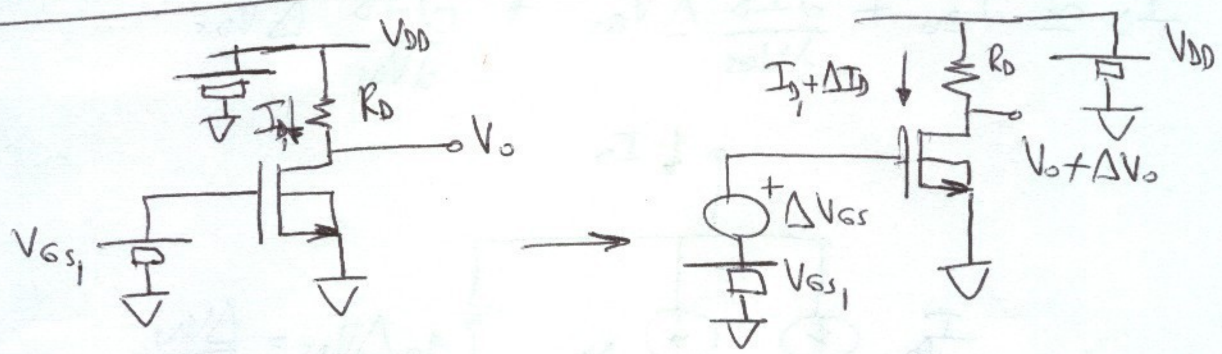
$$R_D^2 I_D^2 - \left[\frac{2}{k'}\frac{L}{W} + 2\,(V_{DD} - V_t)\,R_D\right] I_D + (V_{DD} - V_t)^2 = 0$$

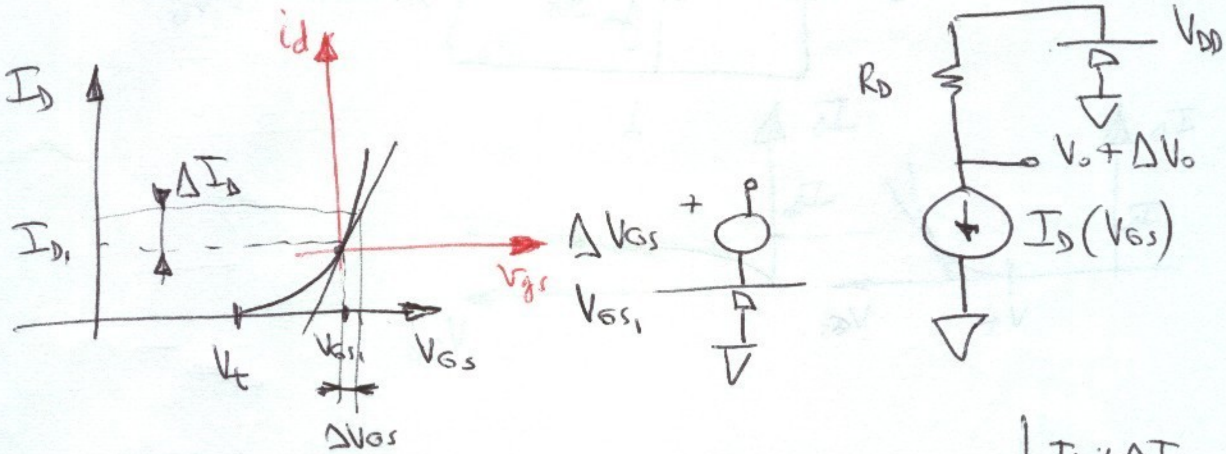
$$25M\; I_D^2 - 110k\; I_D + 81\,V^2 = 0$$

$$I_{D_{1-2}} = \begin{cases} 3.46\,mA \;\times \text{ NOT POSSIBLE} \\ \boxed{935\,\mu A} \rightarrow V_{DS} = V_{GS} = \boxed{5.32\,V} \end{cases}$$

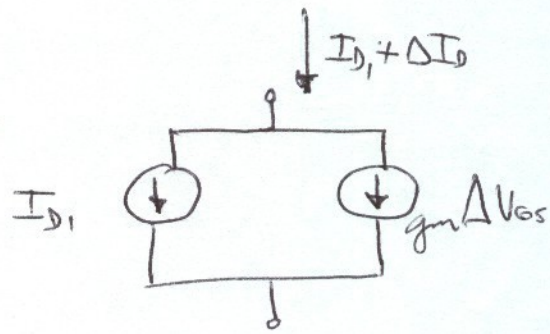


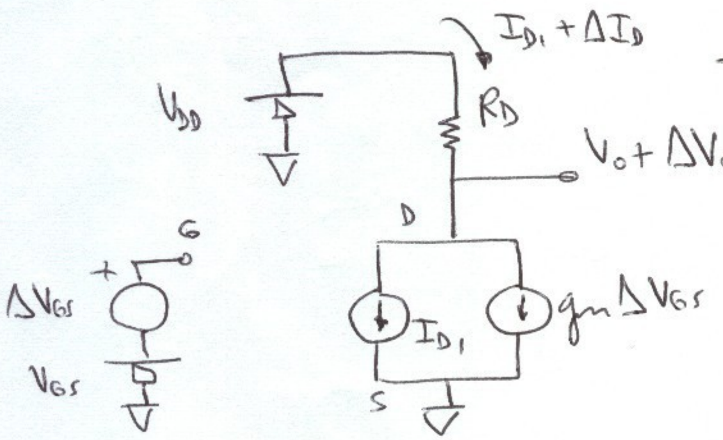
# REVIEW OF SMALL-SIGNAL ANALYSIS



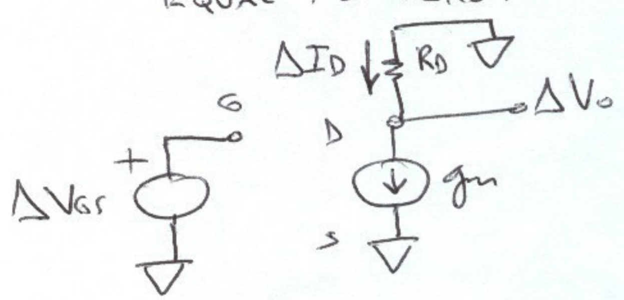

$$I_D \simeq I_{D1} + \underbrace{\underbrace{\left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS1}}}_{g_m} \cdot \Delta V_{GS}}_{\Delta I_D} \rightarrow$$




→ APPLY SUPERPOSITION

- $\Delta V_{GS}=0 \Rightarrow$ DC CIRCUIT BIAS POINT
- EVERYTHING EXCEPT $\Delta V_{GS}$ EQUAL TO ZERO.

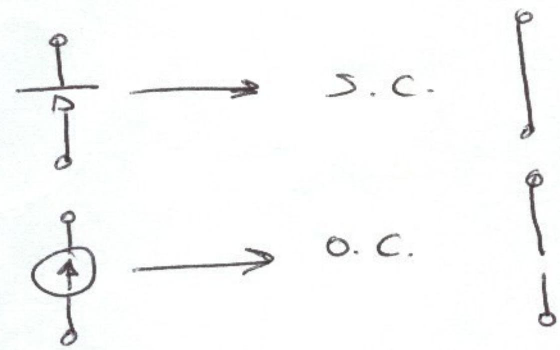



SMALL-SIGNAL CIRCUIT.

# STEPS FOR SMALL-SIGNAL CIRCUIT GENERATION

$\Delta V_{GS}, \Delta I_D, \Delta V_o \longrightarrow v_{gs}, i_d, v_o$ ← LOWERCASE FOR SMALL-SIGNAL QUANTITIES.

- REPLACE NONLINEAR DEVICES WITH LINEARIZED MODELS
- PASSIVATE INDEPENDENT DC SOURCES:




- CURRENTS AND VOLTAGES IN SMALL-SIGNAL CIRCUIT ARE DEVIATIONS FROM DC BIAS POINT. ← PRESENT $\lambda, V_A$ TOO

(PRESENT MOSFET MODEL) AFTER THIS EXAMPLE ↓

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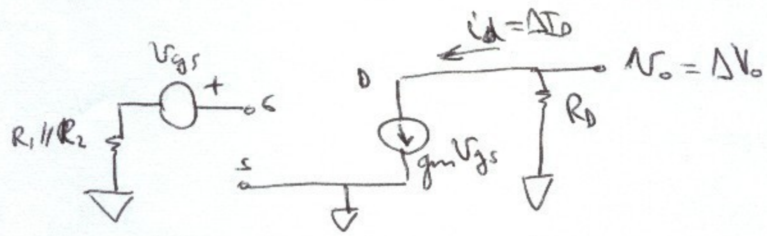
EXAMPLE: CIRCUIT OF EXAMPLE 7.6 WITH $\Delta V_{GS} = v_{gs} = 100\,mV$




$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS}=5V} = k'\frac{W}{L}(V_{GS}-V_t) = 0.4\,mS$$

$$\therefore\ i_d = 0.4\,mS \times 100\,mV = 40\,\mu A$$

$$v_o = 40\,\mu A \times 5\,k\Omega = 0.2\,V$$

TOTAL CURRENT $\cong 800\,\mu A + 40\,\mu A \simeq 840\,\mu A$

EXACT CURRENT $= \frac{k'}{2}\frac{W}{L}(5.1V - 1V)^2 = 840.5\,\mu A$

} quite close