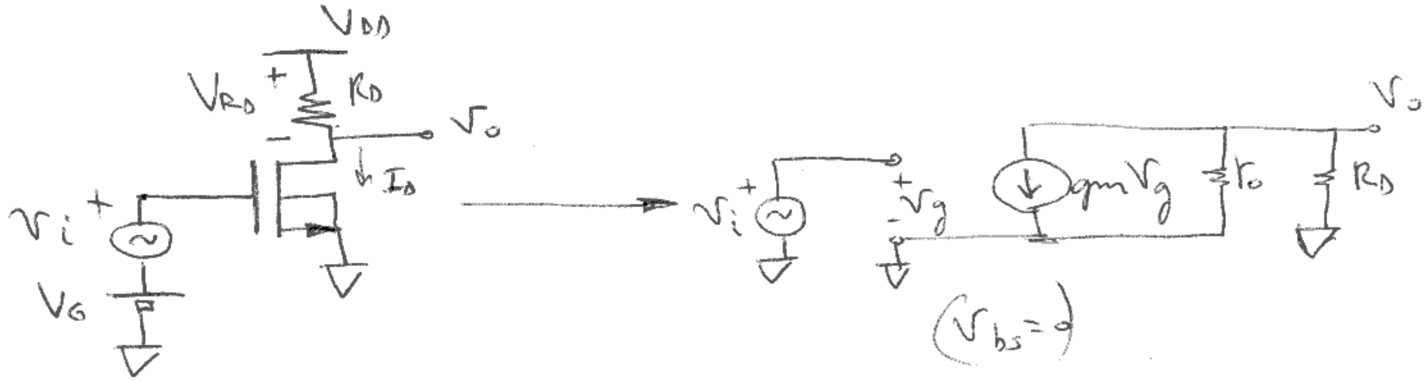
SMALL-SIGNAL ANALYSIS OF CS. AMPLIFIER



$$\frac{v_o}{v_i} = -g_m(r_o // R_D) \longrightarrow \text{ASSUME } r_o \gg R_D$$

$$\frac{v_o}{v_i} \simeq -g_m R_D = -\frac{g_m}{I_D} V_{RD}$$

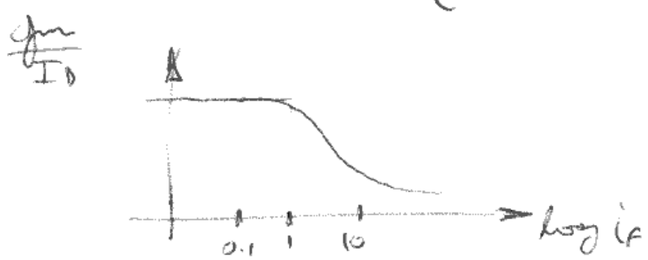
- $V_{R_D} = V_{DD} - V_{DS} \longrightarrow$ MAXIMIZED WHEN $V_{DS} = V_{DSSAT}$
- OTHER THAN V_{RD}, MUST FIND OPTIMUM $\frac{g_m}{I_D}$

$$\frac{g_m}{I_D} = \frac{\frac{2 I_s}{n V_T}\left(\sqrt{1+i_F} - 1\right)}{I_s\, i_F} = \frac{2}{nV_T} \frac{(1+i_F)-1}{i_F(\sqrt{1+i_F}+1)} = \frac{2}{nV_T}\frac{1}{(\sqrt{1+i_F}+1)}$$

W.I) $\frac{g_m}{I_D} \simeq \frac{1}{nV_T}$ CONSTANT

S.I) $\frac{g_m}{I_D} \simeq \frac{2}{nV_T\sqrt{i_F}}$ DECREASES WITH i_F

$\therefore$ MAX GAIN @ W.I. $\begin{cases} \frac{g_m}{I_D} \text{ max} \\ V_{RD} \text{ MAX SINCE } V_{DSSAT} \text{ MIN} \end{cases}$



EXAMPLE: 1.8 V SUPPLY

$I_{SQ} = 300nA$, $n = 1.25$

$\frac{W}{L} = 20$

a) CALCULATE MAX GAIN

$V_{RD\,max} \simeq 1.8V - 100mV = 1.7V$

$\frac{g_m}{I_{D\,max}} \simeq \frac{1}{1.25 \times 26mV} = 30.76\ V^{-1}$

$\left| a_{v\,max} \right| = 52.3$ (NOT MUCH)

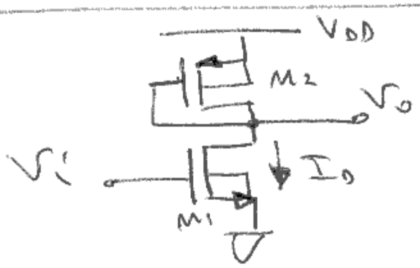
b) CALCULATE R_D:

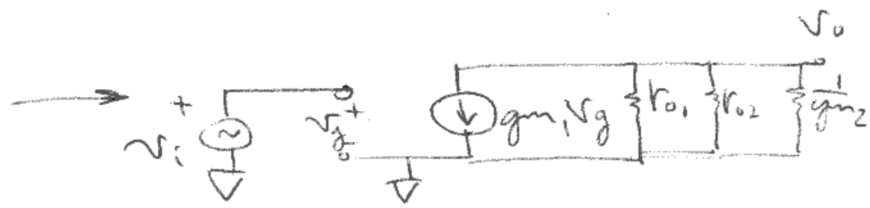
FOR W.I., $i_f \simeq 0.1$ (ASSUME)

$I_D = 300nA \times 20 \times 0.1 = 600nA$

$R_D = \frac{1.7V}{600nA} = 2.83\ M\Omega$ VERY HIGH

DIODE-CONNECTED LOAD





$$\frac{v_o}{v_i} = -g_{m1}\left(r_{o1} \| r_{o2} \| \frac{1}{g_{m2}}\right) \simeq -\frac{g_{m1}}{g_{m2}} = \frac{\frac{2 I_{S1}}{n V_T}\left(\sqrt{1+i_{f1}} - 1\right)}{\frac{2 I_{S2}}{n V_T}\left(\sqrt{1+i_{f2}} - 1\right)}$$

$$\frac{v_o}{v_i} \simeq \frac{n_1 \mu_1 \left(\frac{W}{L}\right)_1}{n_2 \mu_2 \left(\frac{W}{L}\right)_2} \frac{\left(\sqrt{1+i_{f1}} - 1\right)}{\left(\sqrt{1+i_{f2}} - 1\right)}$$

$\rightarrow$ WI) $\frac{n_1 \mu_1 \left(\frac{W}{L}\right)_1}{n_2 \mu_2 \left(\frac{W}{L}\right)_2} \left(\frac{i_{f1}}{i_{f2}}\right)$

$\rightarrow$ S.I) $\frac{n_1 \mu_1 \left(\frac{W}{L}\right)_1}{n_2 \mu_2 \left(\frac{W}{L}\right)_2} \frac{\sqrt{i_{f1}}}{\sqrt{i_{f2}}}$

BETTER:

$$\frac{g_{m1}}{g_{m2}} = \frac{g_{m1}}{I_D} \cdot \frac{I_D}{g_{m2}}$$

$$\frac{v_o}{v_i} \simeq -\frac{2}{n_1}\,\frac{\left(\sqrt{1+i_{f1}}-1\right)}{i_{f1}} \times \frac{n_2\, i_{f2}}{2\left(\sqrt{1+i_{f2}}-1\right)}$$

$$\frac{v_o}{v_i} = -\frac{n_2}{n_1}\,\frac{i_{f2}}{i_{f1}}\,\frac{\left(\sqrt{1+i_{f1}}-1\right)}{\left(\sqrt{1+i_{f2}}-1\right)}$$

W.I) $\frac{v_o}{v_i} \cong -\frac{n_2}{n_1} \simeq 1$ NO GAIN

S.I) $\frac{v_o}{v_i} \cong -\frac{n_2}{n_1}\sqrt{\frac{i_{f2}}{i_{f1}}} = -\frac{n_2}{n_1}\sqrt{\frac{\mu_n n_1 (W/L)_1}{\mu_p n_2 (W/L)_2}}$

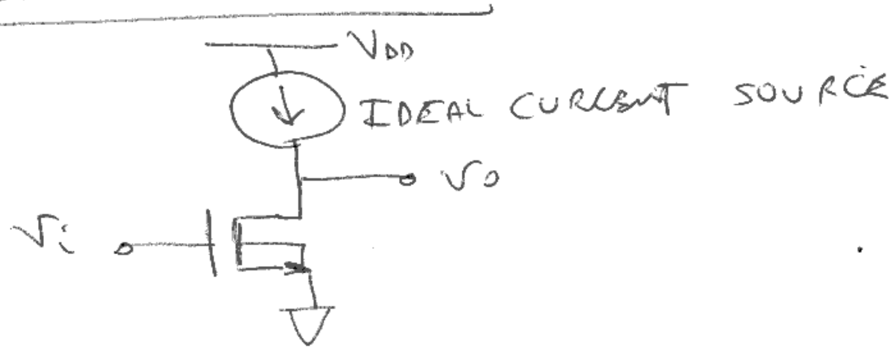
$$= -\sqrt{\frac{\mu_n n_2 (W/L)_1}{\mu_p n_1 (W/L)_2}}$$

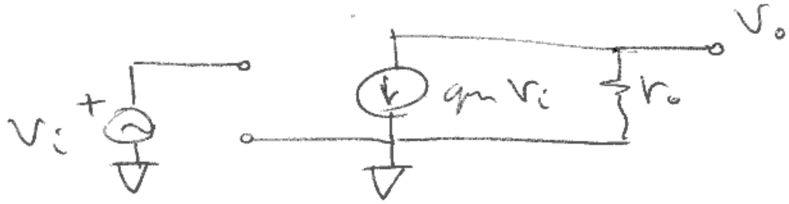
→ GAIN INCREASES IF $i_{f2} \gg i_{f1}$, $\left(\frac{W}{L}\right)_1 \gg \left(\frac{W}{L}\right)_2$ (BUT GAIN SMALL DUE TO $\sqrt{\ }$)

EXAMPLE:
$\mu_n = 400\ \frac{cm^2}{Vs}$
$\mu_p = 160\ \frac{cm^2}{Vs}$
$W_n = 100\mu m$
$W_p = 10\mu m$
$L_n = L_p$, $n_1 = n_2$

$\frac{v_o}{v_i} \simeq -5 \rightarrow i_{f2} \simeq 25\, i_{f1}$

INTRINSIC GAIN STAGE





(CURRENT SOURCE IS AN OPEN CIRCUIT)

$$\frac{v_o}{v_i} = -g_m r_o = -g_m \frac{V_A}{I_D} = -\frac{g_m}{I_D} V_A$$

∴ TO MAXIMIZE GAIN
- MAXIMIZE $\frac{g_m}{I_D}$ (W.I.)
- MAXIMIZE V_A (LONG CHANNEL)

- PROBLEM WITH THAT: FREQUENCY RESPONSE IS BAD.

$$V_A = \frac{L}{\left|\frac{\partial L}{\partial V_{DS}}\right|}$$

EXAMPLE: $\left|\frac{\partial L}{\partial V_{DS}}\right| = 0.02\ \mu m/V$, $L = 2\mu m$, $n = 1.2$

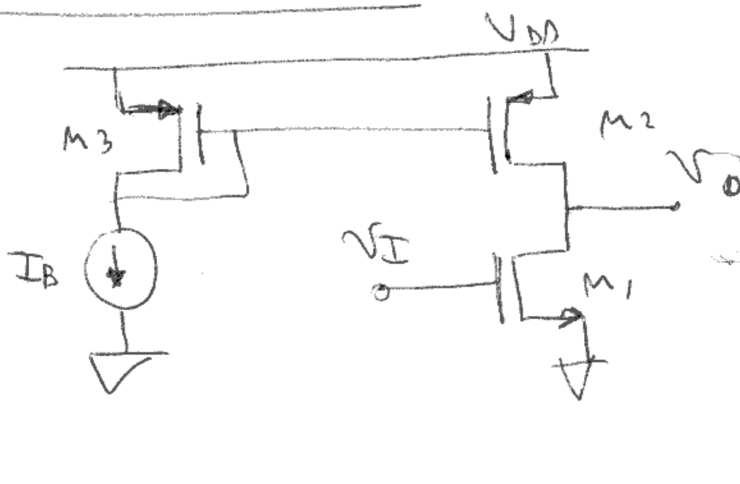
- CALCULATE MAXIMUM INTRINSIC GAIN:

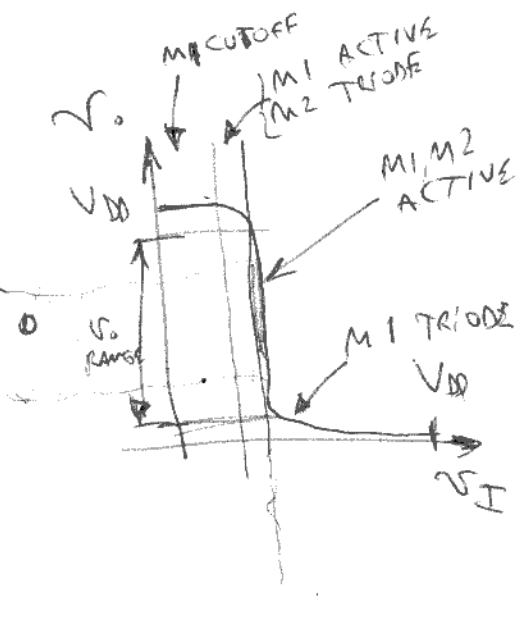
$$\left.\frac{g_m}{I_D}\right|_{max} \simeq 32.05\ V^{-1}$$

$$V_A = 100\ V$$

$$\left.\left|\frac{v_o}{v_i}\right|\right|_{max} = 3205 \rightarrow \text{IN ONE STAGE}$$

CURRENT SOURCE LOAD





- v_o RANGE FOR LARGE GAIN:

$$\underbrace{V_{DS_1 SAT}}_{V_T\left(\sqrt{1+i_{f_1}}+3\right)} \leq v_o \leq V_{DD} - \underbrace{\left|V_{DS_2 SAT}\right|}_{V_T\left(\sqrt{1+i_{f_2}}+3\right)}$$

$$V_T\left(\sqrt{1+\frac{I_B}{I_{S1}}}+3\right) \leq V_O \leq V_{DD} - V_T\left(\sqrt{1+\frac{I_B}{I_{S2}}}+3\right)$$

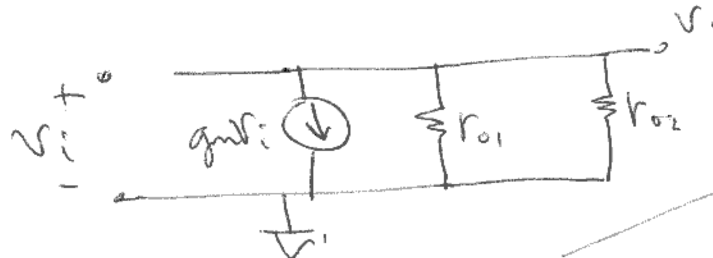
- INPUT VOLTAGE FOR ACTIVE REGION $\Rightarrow I_{D_1} = I_B$

$I_B = I_{S1}\, i_{f_1}$ (i_{r_1} NEGLIGIBLE)

$V_I = ?$ $\quad \frac{V_I - V_{th}}{n} - \underbrace{V_S}_{=0} = V_T\, \mathcal{F}(i_{f_1})$

$$\boxed{V_I = n V_T\, \mathcal{F}\left(\frac{I_B}{I_{f_1}}\right) + V_{th_1}}$$

- SMALL-SIGNAL ANALYSIS:



OBSERVATION: LARGE $I_D \Rightarrow$ {$g_m \uparrow$, $r_o \downarrow$}; SMALL $I_D \Rightarrow$ {$g_m \downarrow$, $r_o \uparrow$}

$$\frac{v_o}{v_i} = -g_m\left(r_{o1} /\!/ r_{o2}\right) = -\frac{g_m}{\left(g_{ds_1}+g_{ds_2}\right)}$$

$$g_{ds_1} = \frac{I_D = I_B}{V_{A1}} \quad g_{ds_2} = \frac{I_{D2}}{V_{A2}} \Rightarrow \frac{v_o}{v_i} = -\frac{g_m}{I_D} \cdot \frac{1}{\left(\frac{1}{V_{A1}}+\frac{1}{V_{A2}}\right)}$$

$$= \frac{2}{nV_T} \frac{1}{\left(\frac{1}{V_{A1}}+\frac{1}{V_{A2}}\right)\left(\sqrt{1+i_f}+1\right)}$$

EXAMPLE:

$V_{A1} = 50V$, $V_{A2} = 30V$, $i_{f_1} = 10$ $\Rightarrow \frac{v_o}{v_i} = -278$

$i_{f_1} = 1 \Rightarrow \frac{v_o}{v_i} = -497$

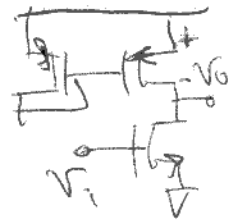
$i_{f_1} = 0.01 \Rightarrow \frac{v_o}{v_i} = -586$ $\qquad i_{f_1} = 100 \Rightarrow -108$

EXAMPLE

- DESIGN A COMMON-SOURCE AMPLIFIER WITH A GAIN OF AT LEAST 150, SLEW RATE @ OUTPUT $\geq 10\frac{V}{\mu s}$ WITH $C_L = 1pF$.

$\left|\frac{\partial L}{\partial V_{DS}}\right|_n = 0.03\frac{\mu m}{V}$ $\quad I_{SQn} = 280\,nA$ $\quad V_{thn} = 0.5V$ $\quad n = 1.25$

$\left|\frac{\partial L}{\partial V_{DS}}\right|_p = 0.04\frac{\mu m}{V}$ $\quad I_{SQp} = 80\,nA$ $\quad V_{thp} = 0.55V$ $\quad n = 1.25$



- $L \geq 1\mu m$ FOR ALL TRANSISTORS (MATCHING REQUIREMENT)
- TRY TO USE LOW AREA. $V_{DD} = 1.5V$

SOLUTION: $\left|\frac{V_o}{V_i}\right| = \frac{2}{nV_T}\frac{1}{\left(\frac{1}{V_{A1}}+\frac{1}{V_{A2}}\right)\left(\sqrt{1+i_{f1}}+1\right)} = 150$

- ASSUME $L = 1\mu m$ TO SAVE AREA $\Rightarrow \begin{cases} V_{A1} = 33.3V \\ V_{A2} = 25V \end{cases}$

$\therefore i_{f1} \leq 2.73$

~~TAKE i_{f1} ...~~

- $I_B \geq C_L\frac{dV_o}{dt} = 10\mu A$ $\qquad \frac{dV_o}{dt} = \frac{I_B}{C_L}$ $\qquad$ (BJT: 384μS)

- TAKE $i_{f1} = 2$ (SAVE AREA), $I_B = 12\mu A$

$\therefore \left(\frac{W}{L}\right)_1 = \frac{12\mu A}{2\times 280\,nA} = 21.4 \simeq 21.5$

$\begin{cases} W_1 = 21.5\mu m \\ L_1 = 1\mu m \end{cases}$

$g_{m1} = \frac{2I_S}{nV_T}\left(\sqrt{1+i_f}-1\right)$ = 0.271 mS
$r_{o1} = 33M\Omega$
$r_{o2} = 2.5M\Omega$
$\frac{1}{g_{m1}} = 3.6k\Omega \ll r_{o1}$

- i_{f2} NOT IMPORTANT FOR GAIN BUT IMPORTANT FOR V_o RANGE

- TAKE $\begin{cases} W_2 = 10\mu m \\ L_2 = 1\mu m \end{cases} \Rightarrow |V_{DSSAT2}| \simeq V_T\left(\sqrt{1+\frac{12\mu A}{10\times I_{SQp}}}+3\right)$

$= 0.182V$.

$V_{DSSAT1} = 123\,mV$

$123mV \leq V_O \leq 1.318V$