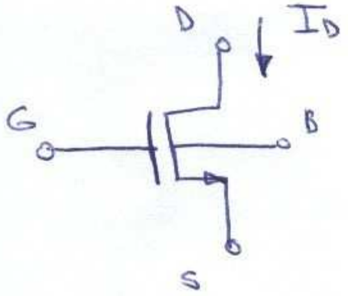
ACM MOSFET MODEL



- ALL VOLTAGES REFERRED TO BULK

$$\begin{cases} V_G = V_{GB} \\ V_D = V_{DB} \\ V_S = V_{SB} \end{cases}$$

- $I_D = \frac{W}{L} I_{SQ} \left(i_f - i_r\right)$
- $I_{SQ} = \frac{n \mu C_{ox} V_T^2}{2}$
- I_{SQ} : SHEET NORMALIZATION CURRENT

n : SUBTHRESHOLD SLOPE FACTOR $(\gtrsim 1.3)$

V_T : THERMAL VOLTAGE

$$V_T = \frac{kT}{q}$$

- i_f, i_r : NORMALIZED FORWARD AND REVERSE INVERSION COEFFICIENTS

$$i_{f,r} \quad \underset{\text{SUBTHRESHOLD}}{\leq 1} \qquad \underset{\text{STRONG INVERSION}}{\geq 100}$$

OTHERWISE MODERATE INVERSION

- PINCH-OFF VOLTAGE $V_P \simeq \frac{V_G - V_{th}}{n}$

$$\frac{V_P - V_S}{V_T} = \mathcal{F}(i_f) = \sqrt{1 + i_f} - 2 + \ln\left(\sqrt{1+i_f} - 1\right)$$

$$\frac{V_P - V_D}{V_T} = \mathcal{F}(i_r)$$

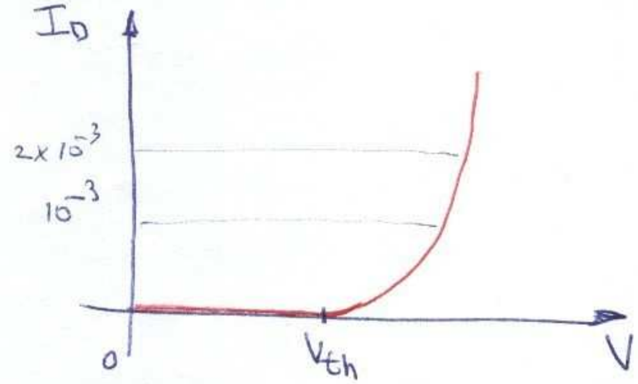
$$\therefore i_{f,r} = \mathcal{F}^{-1}\left(\frac{V_P - V_{S,D}}{V_T}\right)$$

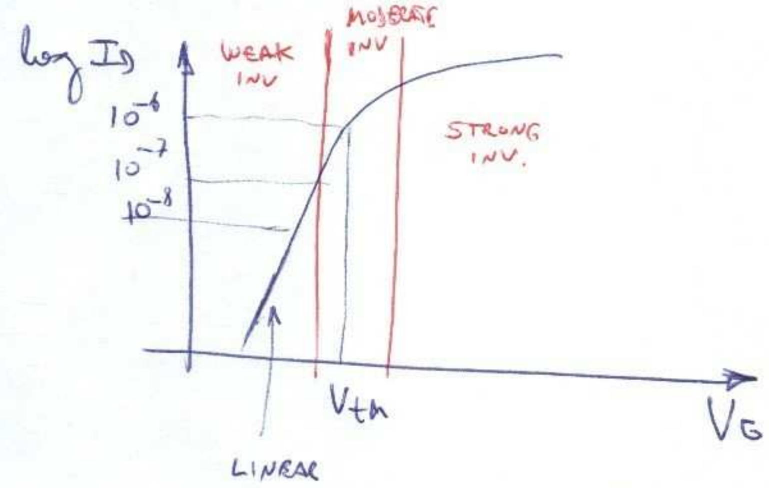
- ADVANTAGES:
  - WORKS FOR ANY INVERSION LEVEL WITH ONE SET OF EQUATIONS.
  - EXPLOITS SYMMETRY OF DEVICE
  - INCLUDES TEMPERATURE DEPENDENCE THROUGH V_{th}, V_T AND μ
  - INCLUDES BODY-EFFECT

~~$V_{th} = V_{th0} - \sigma\,(V_{SB} + V_{DB})$ ↳ DIBL FACTOR, NOT BODY EFFECT~~

- PLOT OF I_D AS A FUNCTION OF V_G $(V_S = 0)$

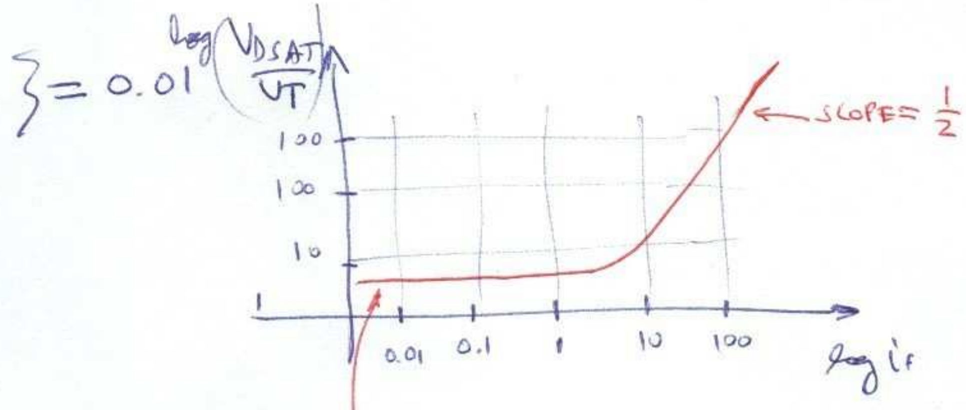
V_{DS} = CONSTANT





$$V_{DSAT} = V_T\left[\ln\left(\frac{1}{\zeta}\right) + \left(1-\zeta\right)\left(\sqrt{1+i_f} - 1\right)\right]$$

ζ : ARBITRARY CONSTANT $\ll 1$



$V_{DSAT} \simeq 4.5 V_T$ FOR WEAK INVERSION

STRONG AND WEAK INVERSION

a) STRONG INVERSION:

(ASSUME $i_r \ll i_f$)

$$\frac{V_P - V_S}{V_T} \simeq \sqrt{1 + i_f} \simeq \sqrt{i_f}$$

$$i_f \simeq \left(\frac{V_P - V_S}{V_T}\right)^2$$

$$I_D = \frac{W}{L} I_{SQ} \left(\frac{V_P - V_S}{V_T}\right)^2$$

$$I_D = \frac{W}{L} \frac{n \mu C_{ox} \cancel{V_T^2}}{2 \cancel{V_T^2}} (V_P - V_S)^2$$

$$I_D = \frac{W}{L} \frac{\mu C_{ox}}{2n} (V_G - V_{th} - nV_S)^2$$

OTHERWISE!

$$I_D \simeq \frac{W}{L} \frac{n\mu C_{ox}}{2} \left[(V_P - V_S)^2 - (V_P - V_D)^2\right]$$

- IF $n = 1 \Rightarrow$ NO BODY EFFECT $\Rightarrow$ QUADRATIC MODEL
- IF $n > 1$ ($n \simeq 1.3$) $\Rightarrow$ BODY EFFECT INCLUDED IN MODEL

NOTE: n IS NOT REALLY CONSTANT (BUT WE'LL ASSUME IT IS CONSTANT)

BODY EFFECT (TRADITIONAL)

$$V_{th} = V_{to} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$$

γ : BULK THRESHOLD PARAMETER ($V^{\frac{1}{2}}$)

ϕ_F : FERMI LEVEL ($\approx 0.3\,V$) $= V_T \ln\left(\frac{N_A}{n_i}\right)$ INVERSION HAPPENS @ $2\phi_F$ (DEPLETION LAYER STOPS GROWING AND CARRIERS ACCUMULATE @ INTERFACE WITH OXIDE)

V_{to} : THRESHOLD FOR $V_{SB} = 0$

b) WEAK INVERSION

$$\frac{V_P - V_S}{V_T} \simeq -1 + \ln\left(\frac{i_f}{2}\right)$$

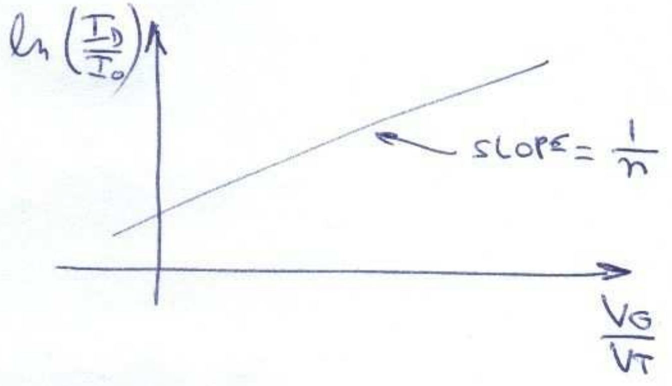
$$i_f \simeq 2\exp\left(\frac{V_P - V_S}{V_T} + 1\right) = 2\, e^{\left(\frac{V_P - V_S}{V_T}\right)} * e^1$$

AGAIN, ASSUME SATURATION $i_f \gg i_r$:

$$I_D \simeq \frac{W}{L} I_{sq}\, 2\exp\left(\frac{V_P - V_S}{V_T}\right) e^1$$

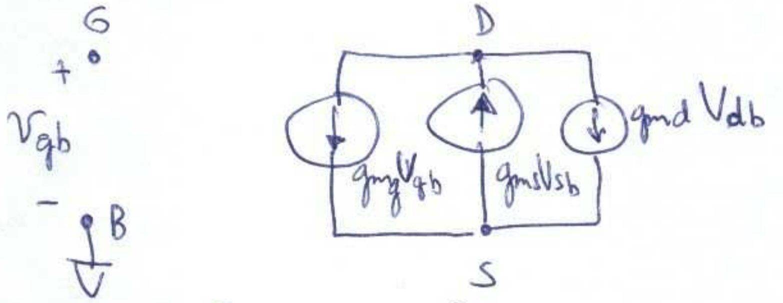
$$I_D = \underbrace{\frac{W}{L} n\mu C_{ox} V_T^2 e^1}_{I_0} \exp\left(\frac{V_P - V_S}{V_T}\right)$$

$\therefore$ I_D GROWS EXPONENTIALLY WITH V_P.



V_S, V_D = CONSTANT

SMALL - SIGNAL MODEL



$$g_{ms} = 2\frac{I_S}{V_T}\left(\sqrt{1 + i_f} - 1\right)$$

$$g_{md} = 2\frac{I_S}{V_T}\left(\sqrt{1 + i_r} - 1\right)$$

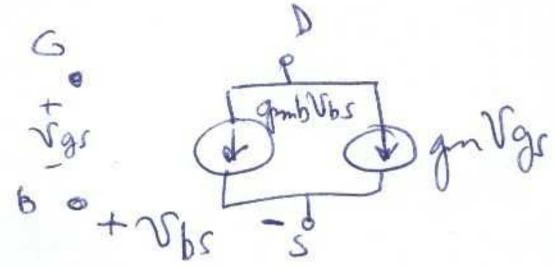
$$g_{mg} = \frac{g_{ms} - g_{md}}{n}$$

DEPEND ON I_S AND n ONLY.

$i_f \gg i_r \Rightarrow$ SIMPLIFY

- EQUIVALENCE WITH TRADITIONAL SMALL-SIGNAL MODEL:

TRADITIONAL:



$$\begin{cases} g_m = \frac{g_{ms}}{n} \\ g_{mb} = g_{ms}\left(\frac{n-1}{n}\right) \end{cases}$$

$$\frac{g_{mb}}{g_m} = X = \frac{n-1}{n} \times n = n-1 \quad (\text{RANGE } 0.1 \sim 0.5)$$

$$n \cong 1 + \frac{\gamma}{2\sqrt{2\phi_F + V_P}}$$

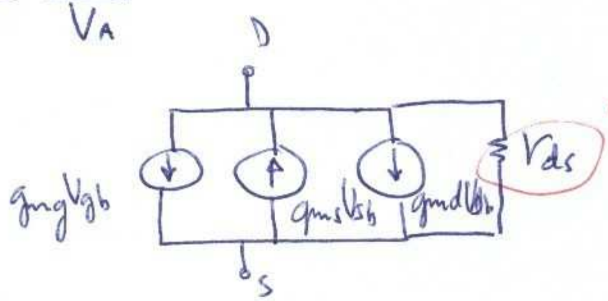
← SOME BOOKS USE V_{SB} BUT V_P IS MORE ACCURATE

OUTPUT CONDUCTANCE

- WE'LL USE THE SAME SIMPLE APPROXIMATION PRESENTED BEFORE:

$$V_A \simeq \left[L\left(\frac{\partial X_d}{\partial V_{DS}}\right)\right]^{-1}$$

$$r_{ds} = \frac{I_D}{V_A}$$



USUALLY NEGLECTED IF DEVICE IN TRIODE REGION

SHORT-CHANNEL EFFECTS

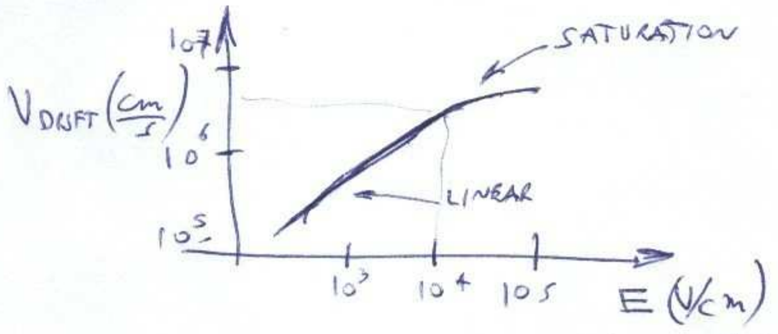
1. EFFECTIVE MOBILITY (NOT REALLY SHORT-CHANNEL EFFECT)

μ DECREASES AS V_G INCREASES $\Rightarrow \mu_{eff}(V_P)$

$$\mu_{eff} = \frac{\mu_0}{1 + \theta\sqrt{V_P + 2\phi_F}} \qquad (11.2.4)$$

2. VELOCITY SATURATION

- DRIFT CARRIER VELOCITY EVENTUALLY SATURATES AND STOPS BEING PROPORTIONAL TO TRANSVERSAL FIELD.



- NET EFFECT IS A REDUCTION IN MOBILITY FOR SHORT-CHANNEL MOSFETS @ HIGHER V_{DS}.

3. CHANNEL-LENGTH MODULATION

- PREVIOUSLY DISCUSSED → PRODUCES INCREASED I_D FOR INCREASING V_{DS}

$$\Delta L = L_c \ln\left(1 + \frac{V_{DS} - V_{DSsat}}{V_p}\right)$$ ← ONE OF THE EFFECTS THAT ORIGINATE V_A

$L_c, \underset{\substack{\uparrow \\ \text{NOT REGULAR } V_P}}{V_p}$: FITTING PARAMETERS

4. DRAIN-INDUCED BARRIER LOWERING (DIBL)

- V_D AND V_S ALSO CONTRIBUTE TO CHANNEL FORMATION FOR SHORT TRANSISTORS ⇒ V_{th} DECREASES WITH V_D, V_S

$$V_{th} \simeq V_{th_{LC}} - \underset{\substack{\uparrow \\ \text{DIBL} \\ \text{FACTOR}}}{\sigma}\left[\left(\underset{\substack{\uparrow \\ \text{SOURCE-BULK} \\ \text{BUILT-IN POTENTIAL}}}{\phi_{bi}} + V_{SB}\right) + \left(\phi_{bi} + V_{DB}\right)\right]$$

(CONTRIBUTES TO V_A)

5. GATE TUNNELING CURRENTS AND BULK CURRENTS

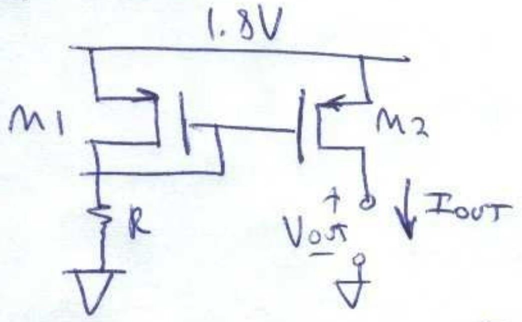
- FOR $t_{ox} < 4\,nm$, ELECTRONS CAN TUNNEL THROUGH OXIDE

∴ LEAKAGE CURRENTS.

- ALSO TUNNELING @ GATE-DRAIN OVERLAP REGION TO BULK : GATE-INDUCED DRAIN LEAKAGE (GIDL)
- IMPACT IONIZATION CURRENTS : ELECTRON-HOLES PAIRS CREATED THAT PRODUCE A D-B CURRENT. DOES NOT HAPPEN IN LOW-VOLTAGE TECHNOLOGIES.

MORE ON CURRENT MIRRORS

EXAMPLE : PROBLEM 5.1



$V_{th} = -0.4V$

$I_S = 1\mu A$

$n = 1.25$

$M1 = M2$

a) CALCULATE R FOR $I_{OUT} = [0.5 \rightarrow 50]\mu A$

b) ″ V_{OUT} MAX

c) $V_A = 5V$, CALCULATE ΔI FOR $V_{OUT} = 0$

d) $V_A = 0V$, ″ ΔI FOR $V_{th_2} = V_{th_1} \pm 10mV$

a)

$$I_{OUT} = I_{D2} \simeq I_S \, i_f \qquad (\text{ASSUME } i_f \gg i_r)$$

$$\therefore \quad i_f = \frac{I_{OUT}}{I_S} = [0.5 \rightarrow 50]$$

$$\underbrace{\frac{V_P}{V_T}}_{V_S=0} = \mathcal{F}(i_f) = \sqrt{1+i_f} - 2 + \ln\left(\sqrt{1+i_f} - 1\right) = [-2.27, 6.96]$$

$$V_P = \frac{V_G - V_{th}}{n} \Rightarrow V_G = (V_{th} + nV_P) = [-0.33V, -0.63V]$$