EELE 5131 : SOLUTIONS TO ASSIGNMENT #1 (2019)

1) $V_{G1} = \frac{R_1}{R_1+R_2} V_{DD} = 2.02\,V$

- ASSUME M1 IN ACTIVE REGION:

$$I_{D1} = \frac{180\,\mu A/V^2}{2} \times 6 \times (2.02V - 0.65V)^2$$
$$= 1.01\ mA$$

- M2 DIODE-CONNECTED $\Rightarrow$ ACTIVE (FOR 'NORMAL' I_D)

$$I_{D2} = \frac{50\,\mu A/V^2}{2} \times 18 \cdot (V_{SG_2} - 0.6V)^2 = 1.01\,mA$$

$$V_{SG_2} = 2.10\,V \Rightarrow V_{DS_1} = \underline{V_{DD} - V_{SG2} = 2.9\,V}$$

- SINCE $V_{DS_1} > V_{OV_1}$ $(= 1.37V)$ $\Rightarrow$ ASSUMPTION OF M1 IN ACTIVE REGION VERIFIED.

$$\boxed{\begin{array}{l} M1 \begin{cases} I_D = 1.01\,mA \\ V_{DS} = 2.9\,V \end{cases} \\ \\ M_2 \begin{cases} I_D = 1.01\,mA \\ |V_{DS}| = V_{SG} = 2.1\,V \quad (V_{DS} = -2.1V) \end{cases} \end{array}}$$

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2) a)

i) $V_{th} \leq v_I \leq V_{DD} \Rightarrow$ CUTOFF $\Rightarrow v_O = 0$

SINCE $I_D = 0$

ii) $V_1 < v_I < V_{DD} - V_{th} \Rightarrow$ ACTIVE

$$v_O = \frac{k'}{2}\frac{W}{L}\left(V_{DD} - v_I - |V_{th}|\right)^2 R_D$$

iii) $v_I < V_1 \Rightarrow$ TRIODE

$$v_O = \frac{k'}{2}\frac{W}{L}\left[2\left(V_{DD} - v_I - |V_{th}|\right)v_{SD} - v_{SD}^2\right] R_D$$

V_1 : EDGE BETWEEN TRIODE AND ACTIVE

$$\underbrace{v_{SD}}_{V_{DD} - v_O} = \underbrace{\left(V_{DD} - v_I - |V_{th}|\right)}_{= V_{OV}}\Bigg|_{v_I = V_1}$$

$$+v_O = +v_I + |V_{th}|$$

$$\frac{k'}{2}\frac{W}{L}\left(V_{DD} - \underset{\substack{\uparrow \\ V_1}}{v_I} - |V_{th}|\right)^2 R_D = \underset{\substack{\uparrow \\ V_1}}{v_I} + |V_{th}|$$

LET $V_1 + |V_{th}| = x$

$$R_D \frac{k'}{2} \frac{W}{L} \left(V_{DD} - x\right)^2 = x$$

$$V_{DD}^2 - 2V_{DD}\, x + x^2 = \frac{1}{\frac{k'}{2}\frac{W}{L} R_D}\, x$$

$$x^2 + \underbrace{\cancel{2V_{DD}}}_{b}\, x + \overbrace{V_{DD}^2}^{=c=} = 0$$

$a = 1$

$b = -6.16\ V$

$c = 9\ V^2$

$a = 1$

$$b = -2V_{DD} - \frac{1}{\frac{k'}{2}\frac{W}{L} R_D}$$

$$x_{1-2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6.16V \pm (1.39\ V)}{2}$$

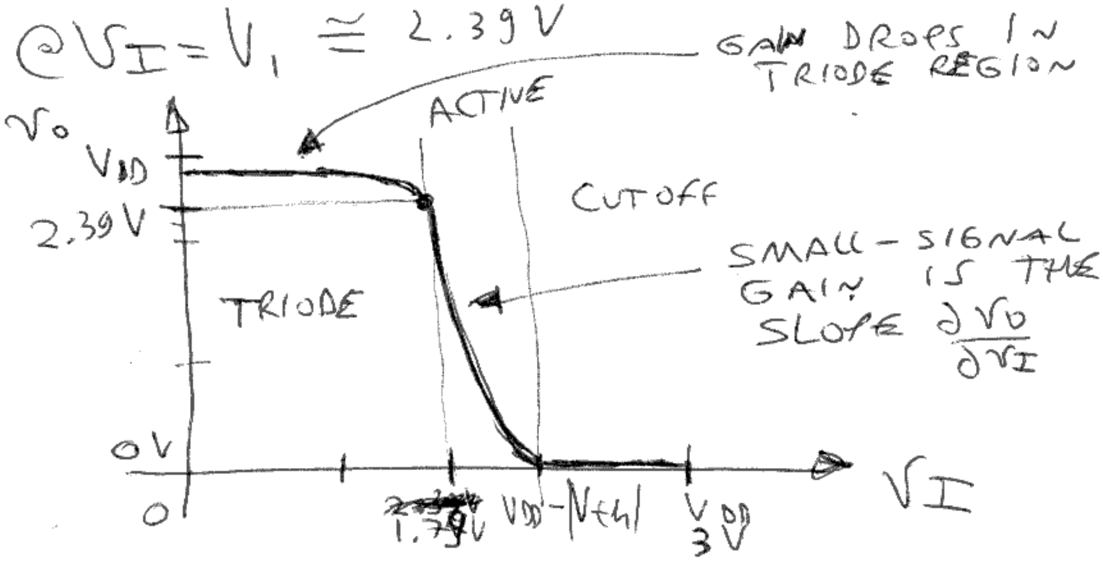
$$\sqrt{b^2 - 4ac} = 1.39\ V$$

$$x_{1-2} = \begin{cases} \cancel{3.76\ V} > V_{DD} + |V_{th}| \Rightarrow M_1 \text{ CUTOFF} \\ 2.39\ V \Rightarrow V_I = x - |V_{th}| = 1.79\ V = V_1 \end{cases}$$

(ROUNDED-OFF, ACTUAL VALUE CLOSER TO 2.38 V)

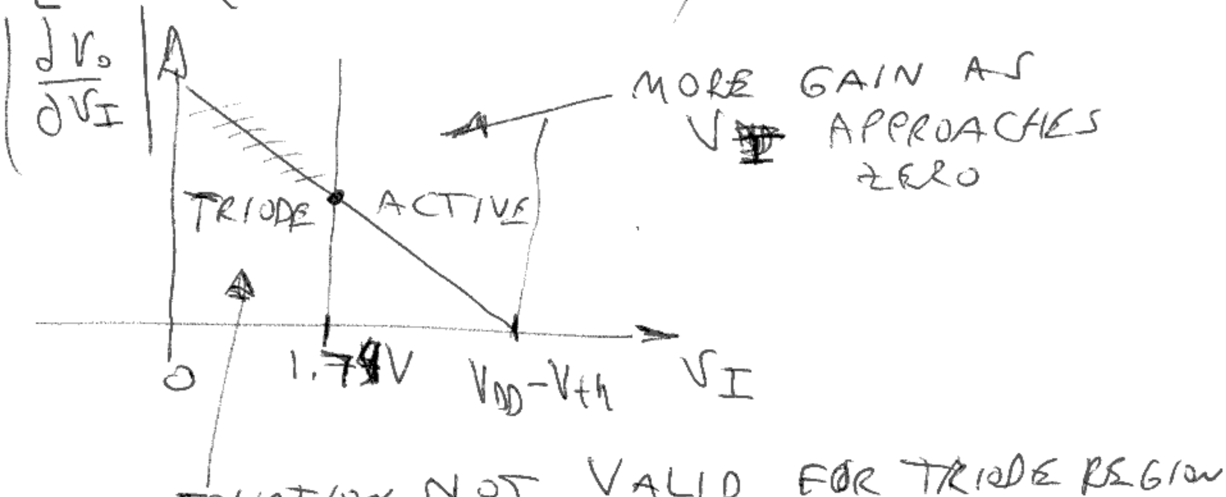
V_o @ $V_I = V_1 \cong 2.39\ V$

b)



④ $$\frac{\partial V_o}{\partial V_I} = \frac{k'}{\cancel{2}}\frac{W}{L}\times\cancel{2}\left(V_{DD}-V_I-|V_{th}|\right)R_D\times(-1)$$

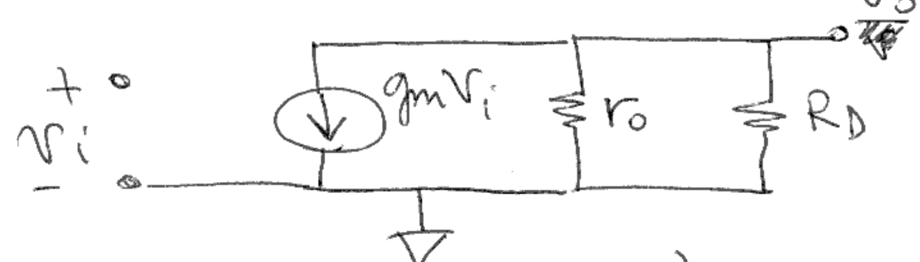
$$\left|\frac{\partial v_o}{\partial v_I}\right| = k'\frac{W}{L}R_D\left(V_{DD}-v_I-|V_{th}|\right)$$



c) $\therefore\ \underline{a_{v_{MAX}} = -7.63}\ \Big|_{@ v_I = 1.79V}$

d) $\Delta v_o = a_v\,\Delta v_I = -7.63\times(-5mV) = \underline{38.15\,mV}$

e)



$V_o = -g_m v_i\,(r_o /\!/ R_D)$

$\frac{V_o}{v_i} = -g_m\,(r_o/\!/R_D)$

$g_m = k'\frac{W}{L}\left(V_{DD}-V_I-|V_{th}|\right) = 0.763\ mS$

$r_o = \frac{L}{I_D\left|\frac{\partial X_D}{\partial V_{DS}}\right|}$, $\quad I_D = \frac{k'}{2}\frac{W}{L}V_{ov}^2 = 0.233\ mA$

$r_o = 86\ k\Omega$

$r_o /\!/ R_D = 8.96\ k\Omega$

$$\boxed{\therefore\ a_v = \frac{V_o}{v_I} = -6.84}$$