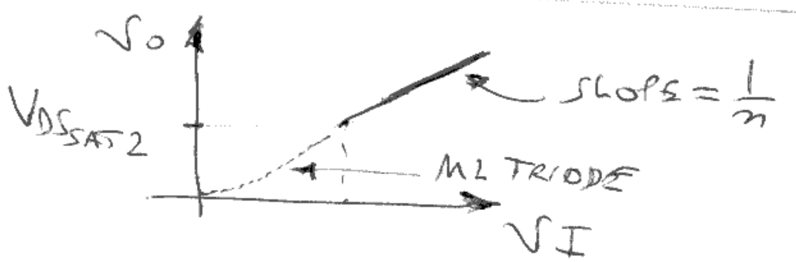
EELE 5131 : SOLUTIONS TO ASSIGNMENT #4 (2019)

1) a)

$$\frac{V_I - V_{th}}{nV_T} - \frac{V_o}{V_T} = F(i_{f1}) \quad (1)$$

$$V_o = \frac{V_I - V_{th}}{n} - V_T F(i_{f1})$$

$$\boxed{V_o = \frac{1}{n} V_I - \left[\frac{V_{th}}{n} + V_T F(i_{f1}) \right]}$$ VALID FOR M_1, M_2 ACTIVE



b)

$$V_{o_{MIN}} = V_{DS_{SAT_2}} = V_T\left(3 + \sqrt{1 + i_{f_2}}\right)$$

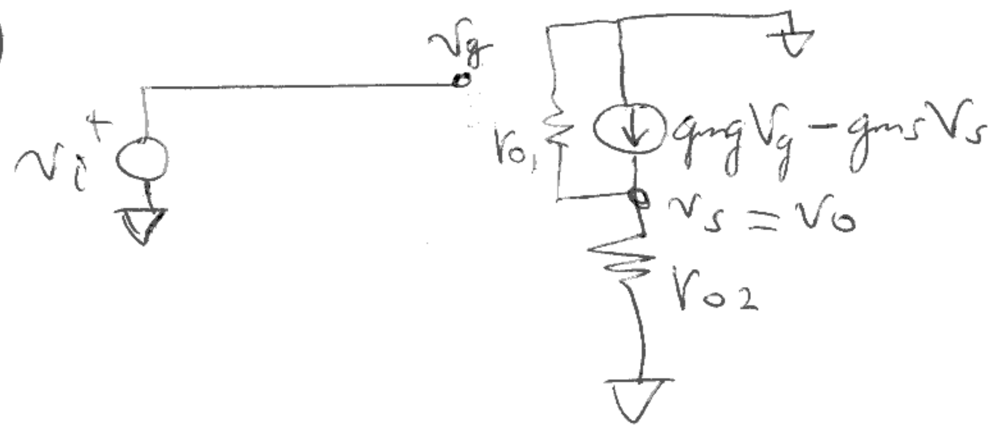
FROM (1) $V_I = V_{th} + nV_o + nV_T F(i_{f1})$

$$\boxed{\therefore V_{I_{MIN}} = V_{th} + nV_{DS_{SAT2}} + nV_T F(i_{f1})}$$

$$V_{o_{MAX}} = V_{DD} - V_{DS_{SAT_1}} = V_{DD} - V_T\left(3 + \sqrt{1 + i_{f1}}\right)$$

$$\boxed{\therefore V_{I_{MAX}} = V_{th} + n\left(V_{DD} - V_{DS_{SAT_1}}\right) + nV_T F(i_{f1})}$$

c)



$$v_o = (g_{mg} v_i - g_{ms} v_o)(r_{o1} // r_{o2})$$

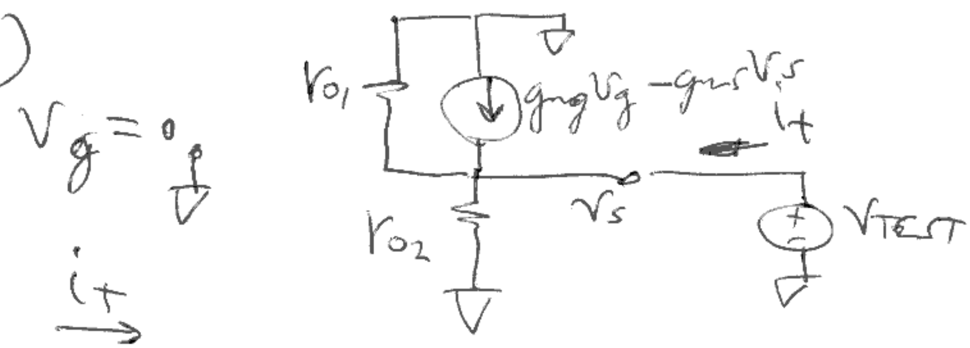
$$v_o (1 + g_{ms}(r_{o1}//r_{o2})) = g_{mg}(r_{o1}//r_{o2})\, v_i$$

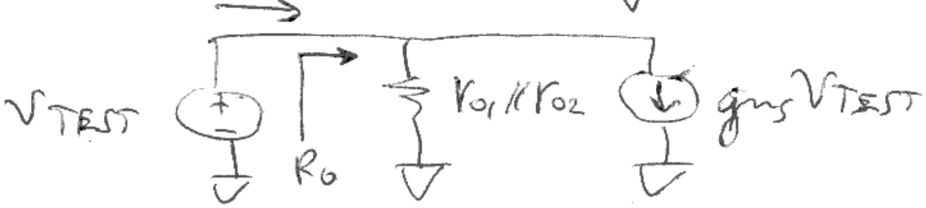
$$\frac{v_o}{v_i} = \frac{g_{mg}(r_{o1}//r_{o2})}{1 + g_{ms}(r_{o1}//r_{o2})}$$

IF $g_{mg}(r_{o1}//r_{o2}) >> 1 \Rightarrow \boxed{\frac{v_o}{v_i} \approx \frac{1}{n}}$

(SINCE $g_{mg} = \frac{g_{ms}}{n}$)

R_o)





$$\boxed{R_o = \frac{V_{TEST}}{i_t} = \frac{1}{\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{ms}} \approx \frac{1}{g_{ms}}}$$

$i_{f_2} = i_{f_3} = \frac{6\mu A}{I_{sq_n} \times 6} = 10$

$r_{o_2} = \frac{1\mu m}{0.07\frac{\mu m}{V} \times 6\mu A} = 2.38 M\Omega$

$i_{f_1} = \frac{6\mu A}{I_{sq_n} \times 15} = 4$

$r_{o_1} = r_{o_2} = 2.38 M\Omega$

$g_{ms} = \frac{2 I_{S1}}{V_T}\left(\sqrt{1+i_f} - 1\right) = 0.143\ mS$

$g_{mg} = 0.110\ mS$

$a_v = \frac{v_o}{v_i} = 0.765$, USING APPROX: $\frac{1}{n} = 0.769$

$R_o = 6952\ \Omega$, USING APPROX $\frac{1}{g_{ms}} = 6993\ \Omega$

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2). $a_v = g_{m_1}(r_{o_1} // r_{o_2}) \geq 100$ (a)

- $V_{DD} - |V_{DSSAT_1}| \geq 1.5V$ (b)

$V_{DSSAT_2} \leq 0.25V$ (c)

- $I_{REF} \geq C_L \times \frac{dV_o}{dt} = 0.5pF \times 50\frac{V}{\mu s}$ (d)

- LOW CURRENT, AREA

- FROM (b), $i_{f1_{MAX}} = ?$

$$\underbrace{V_{DD}}_{=1.8V} - V_T\left(1+\sqrt{1+i_{f1}}\right) \geq 1.5V$$

$$i_{f1} \leq \left(\frac{0.3V}{V_T} - 1\right)^2 - 1 = \underline{110}$$

- FROM (c), $i_{f2_{MAX}} = \left(\frac{0.25V}{V_T} - 1\right)^2 - 1 = \underline{73}$

- FROM (d) $I_{REF_{MIN}} = \underline{25\ \mu A}$

- FROM (a):

$$\frac{2}{n V_T}\frac{1}{\left(\sqrt{1+i_f}+1\right)} \cdot \frac{1}{\frac{\left|\frac{\partial X_D}{\partial V_{DS}}\right|_p}{L_1} + \frac{\left|\frac{\partial X_D}{\partial V_{DS}}\right|_n}{L_2}} \geq 100$$

SINCE $\left|\frac{\partial X_D}{\partial V_{DS}}\right|_n = \left|\frac{\partial X_D}{\partial V_{DS}}\right|_p$ (FOR THIS PROBLEM), MAKE

$L_1 = L_2 = L$

$$\frac{L}{n_p V_T \left|\frac{\partial X_D}{\partial V_{DS}}\right|} \frac{1}{\left(\sqrt{1+i_f}+1\right)} \geq 100$$

$$L \geq 0.24\mu m\left(\sqrt{1+i_f}+1\right) \quad (1)$$

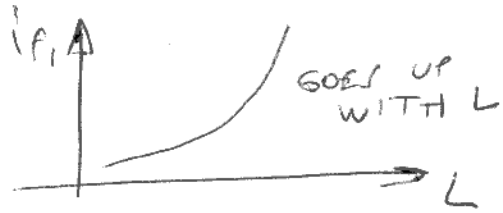
$\therefore i_{f_{MAX}}$ FOR $L = 0.8\mu m$ (LOW AREA) $= \underline{4.44}$

↓ BUT THIS RESULTS IN LARGE W TO CONDUCT 25μA

- IS THERE ANY OPTIMUM L?

AREA $A = W \times L \Rightarrow W = \frac{A}{L} \Rightarrow \frac{W}{L} = \frac{A}{L^2}$

FROM (1) $i_f = \left(\frac{L}{0.24\mu m} - 1\right)^2 - 1$

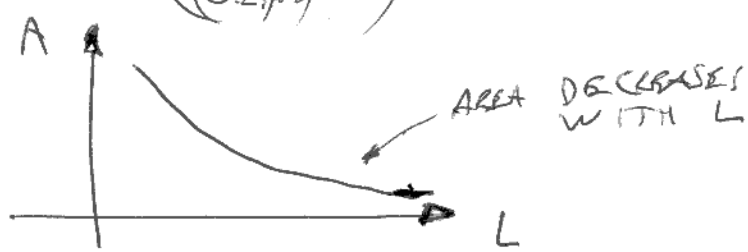


ALSO: $16\mu A = I_{SQP} \frac{W}{L} i_f$ (2)

$= I_{SQP} \underbrace{\frac{A}{L^2}}_{W/L} \underbrace{\left[\left(\frac{L}{0.24\mu m} - 1\right)^2 - 1\right]}_{i_f}$

$$A = \frac{16\mu A}{I_{SQP}} \frac{L^2}{\left(\left(\frac{L}{0.24\mu m}\right) - 1\right)^2 - 1}$$

PLOT A



∴ STRATEGY FOR MIN. AREA: MAKE i_{f_1} CLOSE TO THE MAX $\boxed{i_{f_1} = 100}$ → USE (1) AND FIND L

$\boxed{L = 2.65\mu m}$

↓

GET W FROM (2)

$\boxed{W = 18.5\mu m}$

GATE AREA $\simeq 49.0\mu m^2$

(USING $L = 0.8\mu m \Rightarrow$ GATE AREA $\approx 100\mu m^2$)

$$L_2 = L_1 = 2.65\mu m \Rightarrow W_2 = \frac{16\mu A \times 2.65\mu m}{I_{sq_n} \times 60}$$

SET $i_{f2} = i_{f3} = 60$

$$W_2 = 7.06\mu m$$

FINAL DESIGN:

$$\left(\frac{W}{L}\right)_1 = \frac{18.5\mu m}{2.65\mu m}$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{7\mu m}{2.65\mu m}$$

$$I_{REF} = 16\mu A$$

$i_{f1} \simeq 100$

$i_{f2} \simeq 60$

V_I?

$$|V_{G1}| = V_{DD} - V_I$$

$$|V_{G1}| - |V_{thp}| = nV_T f(i_{f1})$$

$$V_{DD} - V_I = |V_{thp}| + nV_T f(i_{f1})$$

$$V_I = V_{DD} - |V_{thp}| - nV_T f(i_{f1})$$

$$\cong \underline{1.02\,V}$$

3) a) $\underline{I_{D_1} = 10\, I_{D_2}}$ ← DUE TO PMOS MIRROR GAIN → $i_{f_1} = 10\, i_{f_2}$ SINCE $M1 = M2$

$$V_{G1} = V_{th} + n V_T\, F(i_{f_1})$$

$$V_{G2} = V_{th} + n V_{S2} + n V_T\, F(i_{f_2})$$

• SINCE $V_{G1} = V_{G2} \Rightarrow n V_T F(i_{f_1}) = n V_{S2} + n V_T F(i_{f_2})$

$$V_{S2} = V_T\left[F(i_{f_1}) - F(i_{f_2})\right]$$

$$V_{S2} = V_T\left[F(10\, i_{f_2}) - F(i_{f_2})\right]$$ VALID FOR ANY INVERSION LEVEL

• IF M_1, M_2 IN WEAK INVERSION,

$$F(i_f) \simeq \left(\ln \frac{i_f}{2}\right) - 1$$

$$\boxed{V_{S_2} = V_T\left[\ln(10\, i_{f_2}) - \ln(i_{f_2})\right] = V_T \ln 10}$$

$\underline{V_{S2} \text{ IS PTAT}}$

b) $V_{S_2} = 26\text{ mV} \times \ln(10) = \underline{59.9\text{ mV}}$

$\therefore I_{D_2} = \dfrac{V_{S2}}{R} = \dfrac{59.9\text{ mV}}{100\text{k}\Omega} \cong \underline{600\text{ nA}}$

c) $\left.\begin{array}{l} i_{f1} = 0.1 \\ I_{D_1} = 10\, I_{D2} = 6\mu A \end{array}\right\} \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \underline{600}$

$\left.\begin{array}{l} i_{f3} = 0.1 \\ I_{D3} = 600\text{ nA} \end{array}\right\} \left(\frac{W}{L}\right)_3 = \underline{260}$ $\quad \left(\frac{W}{L}\right)_4 = 10\left(\frac{W}{L}\right)_3$

4) $i_{f1} = i_{f2} = 20$

$i_{f3} = i_{f4} = 80$

$i_{f5} = 150$

$L = 2\,\mu m$, $V_{DD} = 1.8V$, $C_L = 0.5\,pF$

a) $V_{IC_{max}} = \underbrace{V_{DD}}_{1.8V} - \underbrace{V_{DSSAT5}}_{0.397V} - \underbrace{V_{GS1}}_{?}$

$$\frac{|V_{G1}| - |V_{thp}|}{n V_T} - \frac{|V_{DSSAT5}|}{V_T} = F(i_{f1})$$

$$|V_{G1}| = |V_{thp}| + n|V_{DSSAT5}| + n V_T F(i_{f1})$$

BUT $|V_{G1}| = -V_{ICM} + V_{DD}$

$$\therefore V_{ICM_{MAX}} = \underbrace{V_{DD}}_{1.8V} - \underbrace{|V_{thp}|}_{=0.43V} - n\underbrace{|V_{DSSAT5}|}_{0.397V} - n\underbrace{V_T F(i_{f1})}_{100\,mV}$$

$$= 0.746\ V$$

b) $V_{ICM} = 0.2V \Rightarrow |V_{G1}| = 1.8V - 0.2V = 1.6V$

$$\frac{|V_{G1}| - |V_{thp}|}{n V_T} - \frac{|V_{DS5}|}{V_T} = F(i_{f1})$$

$$|V_{DS5}| = \frac{|V_{G1}| - |V_{thp}|}{n} - V_T F(i_{f1})$$

$$= 0.786\,V$$

$$V_{O_{MAX}} = \underbrace{V_{DD}}_{1.8V} - \underbrace{|V_{DS5}|}_{0.786V} - \underbrace{|V_{DSSAT2}|}_{0.197V} = 0.817\ V$$

c) $SR = 200\frac{V}{\mu s} \Rightarrow I_{D5} = 0.5pF \times 200\frac{V}{\mu s} = 100\mu A$

$$W_5 = \frac{100\mu A \times 2\mu m}{I_{sqp} \times 150} = 58\mu m$$

$$W_1 = W_2 = \frac{50\mu A \times 2\mu m}{I_{sqp} \times 20} = 217.4\ \mu m$$

$$W_3 = W_4 = \frac{50\mu A \times 2\mu m}{I_{sqn} \times 80} = 12.5\mu m$$