A Universal Parameterized Nonlinear Device Model Formulation for Microwave Circuit Simulation

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Abstract— **A new parameterized nonlinear device model formulation is described that enables the same computer code to be used in any circuit analysis type with no charge conservation issues. The parametric description provides great flexibility for the design of nonlinear device models. The number of parameters or state variables required is the minimum necessary and can be chosen to achieve robust numerical characteristics. An example illustrates charge conservation problems that can occur in the transient simulation of microwave circuits if the models are not correctly formulated.**

I. Introduction

Device model formulation and circuit simulator technology are intricately related with the consequence that for each circuit analysis a particular device formulation is required. For example, one formulation of a transistor model is required for transient analysis and another for harmonic balance. This is required in part because of the need for derivatives in the analysis iteration algorithm but also because of peculiarities related to the choice of state variables and local convergence control. The major contribution of this paper is to present a universal model technology that enables the same model (*i.e.* computer code) to be used with any analysis type; has global convergence properties; and enables physically realistic choice of state variables so that model development can proceed smoothly without the need to use what can be construed as artificial voltage-like or current-like quantities. The use of automatic differentiation also avoids the need to perform derivative evaluations with the device model code dramatically reducing the amount of code needed (typically a factor of 10 reduction is achieved compared to the normal modeling procedure). Object oriented design practices further extend the functionality of device models [1, 2].

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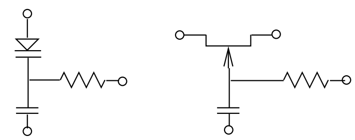

Fig. 1. Topologies that may present charge conservation problems in microwave circuits.

Parameterized models [5, 7] can be used to allow the modeling of nonlinear devices in different analysis types using the same implementation of the device's equations (generic evaluation, [2]). In this way the number of nonlinear state variables is kept reduced to the minimum necessary, the models can be formulated to avoid positive exponential dependencies, and the resulting code can be developed faster and maintained more easily because the equations must be coded only once.

Modern transient circuit simulators use charge or flux as the state variables of nonlinear capacitors or inductors to avoid stability and accuracy problems in transient analysis [3, 6]. This type of problems have rarely been reported for microwave circuits. However they become important when there is a series connection of capacitors and at least one of them is nonlinear [3]. For example, some designs of distributed amplifiers, voltage-controlled oscillators and phase shifters [4] present a serial connection of a Schottky junction with a linear capacitor as shown in Fig. 1.

We will show that in the original parametric formulation it is not always possible to write a charge-conserving model with the minimum number of state variables. Further, we show the necessary modifications to the formulation to obtain a charge-conserving model with the minimum number of state variables and flexible parameterization. The derivation of the Jacobian of the element in the time domain is shown. We apply

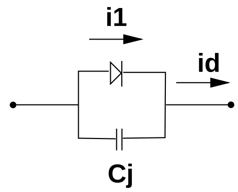

Fig. 2. Schematic of the simplified diode model.

the formulation to a bipolar junction transistor (BJT) using the Gummel-Poon model. The numerical error in transient analysis that results due to a model not based in charge is illustrated with a microwave circuit example.

II. Parameterized Device Models

A nonlinear device model can be described with the following set of equations [7]:

$$\mathbf{v}(t) = \mathbf{v}\left(\mathbf{x}(t), d\mathbf{x}/dt, \ldots, d^m\mathbf{x}/dt^m, \mathbf{x}_D(t)\right) \quad (1)$$
$$\mathbf{i}(t) = \mathbf{i}\left(\mathbf{x}(t), d\mathbf{x}/dt, \ldots, d^m\mathbf{x}/dt^m, \mathbf{x}_D(t)\right) \quad (2)$$

where $\mathbf{v(t)}$ and $\mathbf{i(t)}$ are vectors of voltages and currents at the ports of the nonlinear device, $\mathbf{x}(t)$ is a vector of parameters or state variables and $\mathbf{x}_D(t)$ a vector of time-delayed state variables, i.e., $[\mathbf{x}_D(t)]_i = x_i(t-\tau_i)$. All vectors in Equations (1) and (2) have the same size equal to the number of ports of the nonlinear device being modeled.

Consider the simplified microwave diode model of Fig. 2. The corresponding equations are the following:

$$i_1(v) = I_s(\exp(\alpha v) - 1)$$
$$c_j(v) = \begin{cases} C_{t0}(1 - v/\phi)^{-\gamma} + C_{d0}\exp(\alpha' v) & \text{if } v \leq .8\phi \\ C_{t0}(.2)^{-\gamma} + C_{d0}\exp(\alpha' v) & \text{if } v \geq .8\phi \end{cases},$$

where v is the junction voltage. The capacitor charge q_j can be evaluated as

$$q_j(v) = \int_0^v c_j(u)du.$$

Accurate transient analysis modeling requires q_j to be chosen as the state variable. We need v to calculate $i_1(v)$. Since it is not possible to analytically solve for $v(q)$, the only alternative is to model the diode with two state variables, namely q and v or x (x for the diode model is defined in Ref. [7]). The addition of extra state variables is not desirable because it increases the dimension of the nonlinear system of equations in the circuit analysis algorithm [5].

We propose to formulate the diode equations in different stages. First, the voltage (v) and the current (i_1) through the ideal diode in Fig. 2 are calculated from a parameter x. The capacitor charge $q(v)$ can also be calculated at this stage. Then the current through the capacitor i_c is evaluated at a second stage with

$$i_c = \frac{dq}{dt}.$$

The actual code that performs the derivation is outside the nonlinear device model, and so the model itself is independent of the type of circuit analysis. This procedure provides for good numerical properties of parameterization, charge conservation in transient analysis and immunity to discontinuities in the first derivative of the capacitances. The general formulation is described in the next section.

III. Universal Model Formulation

The nonlinear device models are described by the following set of equations:

$$\text{stage 1} \quad : \quad \begin{cases} \mathbf{f}_1(\mathbf{x}, \mathbf{x}_D) \\ \mathbf{g}_1(\mathbf{x}, \mathbf{x}_D) \end{cases} \quad (3)$$

$$\text{stage 2} \quad : \quad \begin{cases} \mathbf{f}_2(\mathbf{f}_1, d\mathbf{g}_1/dt) \\ \mathbf{g}_2(\mathbf{f}_1, d\mathbf{g}_1/dt) \end{cases} \quad (4)$$

$$\vdots$$

$$\text{stage } n-1 \quad : \quad \begin{cases} \mathbf{f}_{n-1}(\mathbf{f}_{n-2}, d\mathbf{g}_{n-2}/dt) \\ \mathbf{g}_{n-1}(\mathbf{f}_{n-2}, d\mathbf{g}_{n-2}/dt) \end{cases} \quad (5)$$

$$\text{stage } n \quad : \quad \begin{cases} \mathbf{v}(\mathbf{f}_{n-1}, d\mathbf{g}_{n-1}/dt) \\ \mathbf{i}(\mathbf{f}_{n-1}, d\mathbf{g}_{n-1}/dt) \end{cases}. \quad (6)$$

Here $\mathbf{x}$ and $\mathbf{x}_D$ are the state variable vectors defined in Eqs. (1) and (2). The vector functions $\mathbf{f}_j$ and $\mathbf{g}_j$ are evaluated in order of increasing j. The dimension of these vector functions depends on the type of model being implemented. The vectors of voltages and currents, v and i respectively, are evaluated at the end.

This set of equations retain the generality of Eqs. (1) and (2). Elements that originally required only first order derivatives of the state variables now require two function stages. If higher order derivatives were necessary in the formulation (1) and (2), this will translate in several function stages in Eqs. (3)-(6).

The new parametric model formulation shares the advantages of the original one and solves the charge conservation problem in transient analysis. It is also compatible with the generic evaluation technique described

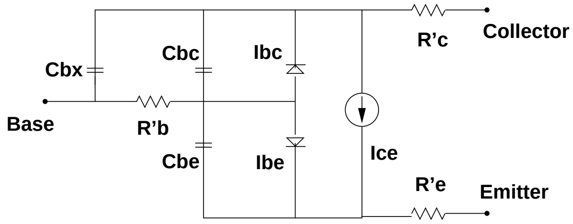

Fig. 3. Bipolar junction transistor model.

in Ref. [2]. This means the nonlinear device models can be described by a unique set of routines used for different circuit analysis types, such as HB or transient analysis. The automatic differentiation technique can be applied to calculate the Jacobian of the nonlinear model because the time differentiation operation is performed outside the model description. The division of the calculation in different stages in Eqs. (3)-(6) also simplifies the implementation of complex device models because it is no longer necessary to express the external currents and voltages from the original state variables and derivatives. Instead, intermediate variables can be calculated and time derivation is applied to some of them.

A. Example: Bipolar Transistor Model

To illustrate universal model formulation consider the simplified NPN-type BJT model of Fig. 3. The voltages across the base collector capacitor v_{bc} and base emitter capacitor v_{be} are chosen as state variables (a similar parameterization as explained in Ref. [7] can also be done). Note that we only need two state variables. If the charge at the capacitors where chosen, this number would be three and would thus increase the dimension of the nonlinear system of equations in the simulation algorithm [5]. With the proposed approach we need three functional stages to model the transistor. Each stage has two input and two output parameters. In stage 1 the dc current components I_{bc} and I_{be} are computed as

$$\begin{aligned} I_{be} &= \frac{Ibf}{\beta_F} + Ile \\ I_{bc} &= \frac{Ibr}{\beta_R} + Ilc, \end{aligned}$$

where Ibf and Ile are the components of the currents through the base-emitter diode and Ibr and Ilc are components of the the currents through the base-collector diode. The first output vector from stage 1 stores the charge across the base collector and base emitter capacitors which can be evaluated as

$$\begin{aligned} q_{bc}(v) &= \int_0^v c_{bc}(v_{bc}) dv_{bc} \\ q_{be}(v) &= \int_0^v c_{be}(v_{be}) dv_{be}. \end{aligned}$$

The second output vector stores the diode current components and junction voltages. The derivative of the charge across the capacitors,

$$\begin{aligned} I_{Cbc} &= \frac{dq_{bc}}{dt} \\ I_{Cbe} &= \frac{dq_{be}}{dt}, \end{aligned}$$

is obtained as an input parameter in stage 2. The above result is used to calculate the charge across the distributed base collector capacitor C_{bx}.

Inputs to stage 3 contain the corresponding current through C_{bx} and the junction voltages. In stage 3 the final external voltages and currents are calculated using the intermediate variables generated at the previous stages.

B. Time Domain Jacobian

In the time domain, the time derivatives are approximated by a function $h(x_i)$ that depends on the current and previous history of the variable to be derived. Similarly the elements of the time-delayed state variable vector $\mathbf{x}_D$ is calculated by a function $k(x_i)$:

$$\begin{aligned} \frac{dx_i}{dt} &\approx h(x_i) \\ x_i(t-\tau) &\approx k(x_i). \end{aligned}$$

The Jacobians for $\mathbf{f}_1$ and $\mathbf{g}_1$ are then

$$\begin{aligned} \mathbf{J}_{f_1} &= \mathbf{J}_{f_1,x} + \mathbf{J}_{f_1,x_D}\, d\mathbf{k}(\mathbf{x}) \\ \mathbf{J}_{g_1} &= \mathbf{J}_{g_1,x} + \mathbf{J}_{g_1,x_D}\, d\mathbf{k}(\mathbf{x}), \end{aligned}$$

where $d\mathbf{k}(\mathbf{x})$ is a diagonal matrix where each diagonal element is calculated as $dk(x_i)/dx_i$. The diagonal matrix $d\mathbf{h}(\mathbf{x})$ is similarly defined. The Jacobian matrices of the form $\mathbf{J}_{y,z}$ are obtained directly from the automatic differentiation routines, *i.e.* they do not need to be coded explicitly.

The Jacobians for $\mathbf{f}_n$ and $\mathbf{g}_n$ are calculated by

$$\begin{aligned} \mathbf{J}_{f_n} &= \mathbf{J}_{f_n,f_{n-1}} \mathbf{J}_{f_{n-1}} + \mathbf{J}_{f_n,g_{n-1}}\, d\mathbf{h}(\mathbf{g}_{n-1}) \mathbf{J}_{g_{n-1}} \\ \mathbf{J}_{g_n} &= \mathbf{J}_{g_n,f_{n-1}} \mathbf{J}_{f_{n-1}} + \mathbf{J}_{g_n,g_{n-1}}\, d\mathbf{h}(\mathbf{g}_{n-1}) \mathbf{J}_{g_{n-1}}. \end{aligned}$$

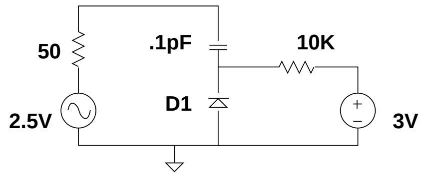

Fig. 4. Varactor circuit.

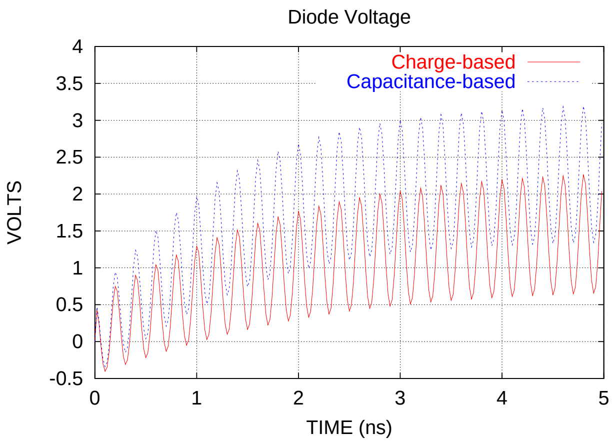

Fig. 5. Comparison of the simulations using the capacitor- and the charge-based models.

The final Jacobian matrices $\mathbf{J}_u$ and $\mathbf{J}_i$ are obtained in a similar way. It is important ot remark that this calculation of the Jacobian is the same for any element and thus it can be implemented outside the actual element routine.

IV. Discussion

Here we present an illustration of the dramatic loss of accuracy that can result from nonconservation of charge taking the circuit of Fig. 4 as an example. The simulations were performed with the *Transim*[1] program. Fig. 5 compares two identical transient simulation results using the diode model with a capacitor-based model and with a charge-based model. Table I shows that reducing the time step tends to reduce the error due to nonconservation of charge. This type of accumulation of numerical error is not present in other analysis types. For example the harmonic balance simulation produces exactly the same result with either diode model.

[1]http://guppie.ncsu.edu/transim/

TABLE I

Comparison of the numerical error due to nonconservation of charge as a function of the time step.

Time Step (ps)	Numerical Error (V)
20	.917
10	.533
5	.295
2.5	.161

V. Conclusions

A new parameterized nonlinear device model formulation has been presented. This formulation enables a unique description of the model in any circuit analysis type and provides a mechanism to describe complex charge-based models without adding extra state variables to the nonlinear system of equations in the main simulation algorithm. In some cases it also simplifies the equations that must be coded in the model implementation. The parametric description provides great flexibility for the design of nonlinear device models. The number of parameters or state variables required is the minimum necessary and they can be chosen to achieve robust numerical characteristics. We have demonstrated the numerical errors that may arise from a capacitance-based nonlinear device model in the transient simulation of microwave circuits. The necessary support for the description of nonlinear elements using the new formulation has been implemented in the Transim circuit simulator.

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