HARMONIC BALANCE ANALYSIS FOR SYSTEMS WITH CIRCUIT-FIELD ITERATIONS

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ABSTRACT

A state variable implementation of harmonic balance circuit analysis for spatially distributed systems is developed. The equations are formulated with the minimum number of unknowns and error functions starting from the modified nodal admittance matrix of the linear part of the circuit. The program uses advanced numerical techniques, such as automatic differentiation to calculate the Jacobian of the system and it is compatible with port-based network descriptions which occur with electromagnetic modeling.

INTRODUCTION

Steady state (harmonic balance) analysis continues to be the dominant approach to the simulation of nonlinear RF and microwave circuits. Development efforts are currently being directed at extending the approach to accommodate a very large number of tones, improve robustness, and developing iterative matrix-free methods to handle very large problems without explicit storage of the Jacobian. One system that is of particular concern is the simulation of quasi-optical power combiners with hundreds of active devices with significant thermal effects and distributed over an electrically large region. This exacerbates the problem of robustness and of model development. Electromagnetic analysis is essential in modeling electrically large structures and this results in a characterization in terms of ports.

This paper develops a robust harmonic balance procedure compatible with a port-based network description.

Model development is greatly simplified using a novel implementation of automatic differentiation technology.

NONLINEAR EQUATION FORMULATION

In this section the harmonic equations are formulated with the minimum number of unknowns and error functions starting from a modified nodal admittance matrix of the linear part of the circuit. This approach has the advantage that the flexibility of the modified nodal admittance matrix is kept together with the robustness advantage given by the state variable approach.

The formulation of the system equations begins with the partitioned network of Fig. 1, with the nonlinear elements replaced by variable voltage or current sources [2]. For each nonlinear element one
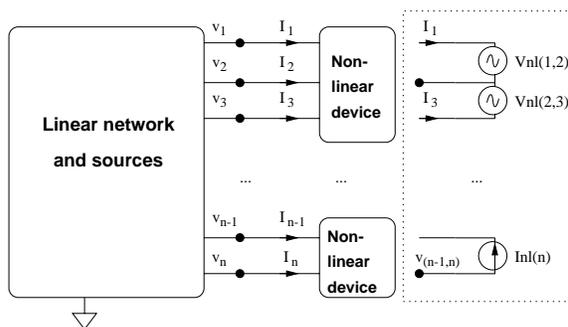

Fig. 1. Network with nonlinear elements.

terminal is taken as the reference and the element is replaced by a set of sources connected to the reference terminal. Clearly, the state of the element can

be determined considering only the current of the sources, or the voltages across the sources, or some combination of the voltages and currents. Identifying the local element reference eliminates one potential state variable and one error function term that would otherwise be considered if all of the nonlinear element terminals were treated equally. For example, in a conventional voltage-current formulation the voltage and current associated with the reference node need not be considered in the nonlinear formulation. With the three-terminal element in Fig. 1, terminal 2 is chosen as the reference node and only two voltages, $V_{12} = (V_1 - V_2)$ and $V_{32} = (V_3 - V_2)$, and two currents, I_1 and I_3, are needed.

Using the state variable concept, let $\mathbf{X}$ be the *state variable vector*. Each element of $\mathbf{X}$ is a vector with all the frequency components of the same state variable, thus

$$\mathbf{X} = \begin{bmatrix} x_{1,f0} & x_{1,f1} & x_{1,f2} & \cdots & x_{1,fm} \\ x_{2,f0} & x_{2,f1} & x_{2,f2} & \cdots & x_{2,fm} \\ \vdots & & & & \\ x_{n,f0} & x_{n,f1} & x_{n,f2} & \cdots & x_{n,fm} \end{bmatrix} \quad (1)$$

We denote X_i as the i th row of the matrix, i.e., all the frequency components of the i th state variable.

Now the equations can be formulated. Unlike[1], here the equations are formulated in the frequency domain. The current and voltage at the i th terminal of each nonlinear device are

$$V_i = V_i(X_k, \ldots, X_l) \quad (2)$$
$$I_i = I_i(X_k, \ldots, X_l) \quad (3)$$

where the element under consideration depends only on the k th to l th components of the state variable vector. Since the state variable approach is used, both the current and the voltage of the nonlinear elements are needed for developing the error function of the network.

Since the nonlinear elements are replaced by sources, the modified nodal admittance matrix (MNAM) of the entire circuit can be written for each frequency:

$$\mathbf{M}_{fi}V_{fi} = S_{fi} \quad (4)$$

The source vector S_{fi} is composed of the fixed sources present in the circuit and the state variable-dependent sources due to the nonlinear elements. For simplicity, assume that all the nonlinear devices are replaced by current sources. Let $I_{NL,fi}(X)$ be the vector with the currents of all the nonlinear elements at frequency f_i. These currents are 'applied' to the linear circuit. Then, the source vector can be written as:

$$S_{fi} = S_{\text{fixed},fi} + \mathbf{W}I_{NL,fi}(\mathbf{X}) \quad (5)$$

Where $\mathbf{W}$ is a matrix that maps the nonlinear currents to the proper place in the source vector. This matrix is composed only of zeros, ones and minus ones, and it is the same for all frequencies since it only contains topology information.

Provided that the MNAM is not singular, we can calculate the voltages at the linear circuit given the values of the state variables $\mathbf{X}$:

$$V = \mathbf{M}_{fi}^{-1} S_{\text{fixed},fi} + \mathbf{M}_{fi}^{-1}\mathbf{W}I_{NL,fi}(\mathbf{X}) \quad (6)$$

We denote $U_{NL,fi}(\mathbf{X})$ the vector with the voltages of all the current sources due to the nonlinear elements at frequency f_i (port voltages of the nonlinear elements). The error function is formulated by comparing the port voltages of the nonlinear elements $U_{NL,fi}(\mathbf{X})$ with the port voltages at the linear circuit $U_{L,fi}(\mathbf{X})$. In order to obtain $U_{L,fi}(\mathbf{X})$, we need to perform differences between the nodal voltages of the current source's terminals. This can be accomplished by multiplying the nodal voltage vector V by a matrix $\mathbf{T}$ such that the result are the desired port voltages. It can be shown that $\mathbf{W}$ is the transpose of $\mathbf{T}$ so that:

$$U_{L,fi}(\mathbf{X}) = \mathbf{T}\mathbf{M}_{fi}^{-1} S_{\text{fixed},fi} + \mathbf{T}\mathbf{M}_{fi}^{-1}\mathbf{T}^T I_{NL,fi}(\mathbf{X}) \quad (7)$$

The first term in this sum is a constant vector, and the second is a constant matrix multiplied by

a function of $\mathbf{X}$. Then,

$$
\begin{aligned}
S_{sv,fi} &= \mathbf{T}\mathbf{M}_{fi}^{-1}S_{\text{fixed},fi} && (8) \\
\mathbf{M}_{sv,fi} &= \mathbf{T}\mathbf{M}_{fi}^{-1}\mathbf{T}^{T} && (9)
\end{aligned}
$$

$S_{sv,fi}$ is the state variable source vector, and the matrix $\mathbf{M}_{sv,fi}$ is the state variable impedance matrix.

The solution of the problem is achieved by finding the zero of each error function

$$
F_{fi}(\mathbf{X}) = U_{L,fi}(\mathbf{X}) - U_{NL,fi}(\mathbf{X}) = 0 \qquad (10)
$$

The complete system of equations is then

$$
\begin{aligned}
F(\mathbf{X}) &= \\
&\begin{bmatrix}
S_{sv,f0} + \mathbf{M}_{sv,f0}I_{NL,f0}(\mathbf{X}) - U_{NL,f0}(\mathbf{X}) \\
S_{sv,f1} + \mathbf{M}_{sv,f1}I_{NL,f1}(\mathbf{X}) - U_{NL,f1}(\mathbf{X}) \\
\vdots \\
S_{sv,fm} + \dots
\end{bmatrix} \\
&= \mathbf{0} \qquad\qquad\qquad\qquad\qquad\qquad\qquad (11)
\end{aligned}
$$

The resulting system of equations is composed of $m+1$ systems each with n equations. If f_0 is denoted as being DC, then the dimension of the system (and the number of unknowns) is $(2m+1)n$, since for each state variable a real component is needed for DC, and a real and imaginary component for each alternating frequency. This is the minimum number of unknowns that can be achieved, and it only depends on the nonlinear elements and the frequencies considered.

After solving (11), the value of the state variable vector is known, so finding the voltages (and the current of the ideal voltage sources) for the entire network is straightforward using (6).

Note that the entire analysis was performed in the frequency domain. All the frequency mixing is performed in the functions of (2), i.e., inside the nonlinear devices. The particular method used to calculate the currents and voltages from the state variables in the frequency domain is not relevant here.

IMPLEMENTATION

The goal in designing the program was to allow speed in development, to use "off the shelf" advanced numerical techniques, and to allow easy expansion and testing of new models and numerical methods.

A. Modified nodal admittance matrix representation

For the MNAMs, the package *Sparse 1.3* [5] is used. It is a flexible package of subroutines written in C that quickly and accurately solves large sparse systems of linear equations. It also provides utilities such as MNAM reordering, etc. There is one sparse matrix per frequency. The linear system solving capability is used to calculate the matrices in (8) and (9).

B. Frequency-time conversion

Each nonlinear element has a function that given the value of its state variables in the frequency domain, returns the phasors of the currents and voltages at its ports. The actual calculation is performed in the time domain, using the FFT to convert between time and frequency domain. Multitone analysis is supported by using artificial frequency mapping [6]. This approach is more efficient than the multidimensional Fourier transform, and it is simpler to implement.

C. Solution of the nonlinear system

The nonlinear system is solved using the library *NNES* [3]. It is written in Fortran and it provides Newton and quasi-Newton methods with many options, such as the use of analytic Jacobian or forward, backwards or central differences to approximate it, different quasi-Newton Jacobian updates, two globally convergent methods, etc.

D. Automatic differentiation

The analytic Jacobian is calculated in the routine using *Adol-C* [4]. This is a software package written in C and C++ that performs automatic
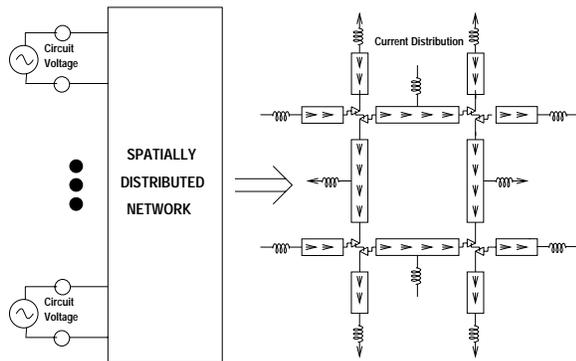
Fig. 2. Layout for 2x2 quasi-optical grid amplifier with bias inductors shown.

Fig. 3.   Power output for 2x2 grid amplifier system: solid line, measurement; dashed line, simulation.

differentiation. The numerical values of derivative vectors are obtained free of truncation errors at a small multiple of the run time and randomly accessed memory of the given function evaluation program.

SIMULATION OF A QUASI-OPTICAL GRID

The system modeled is shown in Figure 2 [7]. The input is a normally incident, horizontally polarized wave and the output is normal but vertically polarized. The quasi-optical grid was modeled by a multi-port admittance matrix [8] derived from a electromagnetic simulator.

The simulated and measured output power is shown in Fig. 3.

CONCLUSIONS

An state variable-based harmonic balance analysis was developed for the simulation of spatially distributed circuits, especially for quasi-optical circuits. The equation formulation used allows the simulation of quasi-optical systems transparently, without ground problems and redundancies presented by other formulations.

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[1]   V. Rizzoli, A. Lipparini, A. Costanzo, F. Mastri, C. Ceccetti, A. Neri and D. Masotti, *State-of-the-Art Harmonic-Balance Simulation of Forced Nonline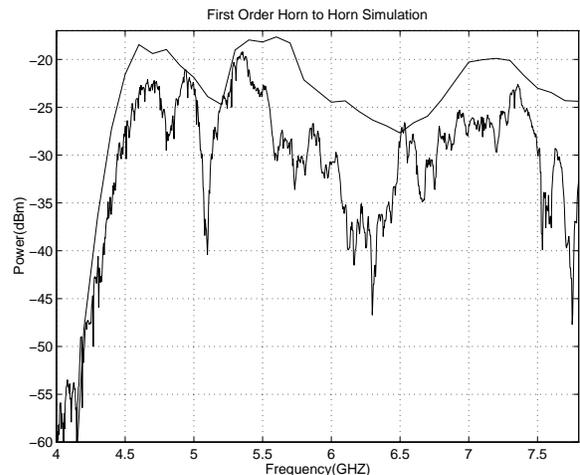ar Microwave Circuits by the Piecewise Technique*, IEEE Trans. on Microwave Theory and Techniques, Vol. 40, No. 1, Jan 1992.

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