Generalized Circuit Formulation for the Transient Simulation of Circuits using Wavelet, Convolution and Time-Marching Techniques

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Abstract — **This paper outlines a general method to analyze circuits with the minimum number of nonlinear unknowns and error functions. For some circuits, this number is much smaller than the size of the nonlinear system resulting from applying conventional formulations. The linear subnetwork is described by a modified nodal admittance matrix. Robustness is obtained by a state-variable approach. We show the derivation of different transient analysis equations based on wavelets, convolution and time-marching techniques from a general set of equations.**

1 Introduction

The most widespread method of nonlinear circuit analysis is time-domain analysis (also called transient analysis) using programs like Spice. Such programs use numerical integration to determine the circuit response at one instance of time given the circuit's response at a previous instance of time. However, it is not always practical to simulate analog circuits using numerical integration. Application of nonlinear analysis using domains other than the time domain is a current area of research. There are many other ways of thinking about circuit analysis. Some are ideally suited to particular applications. An example is *harmonic balance* (HB) analysis of RF and microwave circuits. Harmonic balance differs from traditional time domain methods in that time domain simulators represent waveforms as a collection of samples whereas harmonic balance represents them using the coefficients of the sinusoids. Rizzoli [1] proposed a state variable approach in the HB simulation context which provides great flexibility for the design of nonlinear device models. The state variables can be chosen to achieve robust numerical characteristics. In addition, the flexibility of the state variable formulation allows the generic evaluation mechanism described in Reference [2] to be implemented. The authors of this paper presented a formulation of the HB analysis based the state variable concept using modified nodal analysis in Reference [3].

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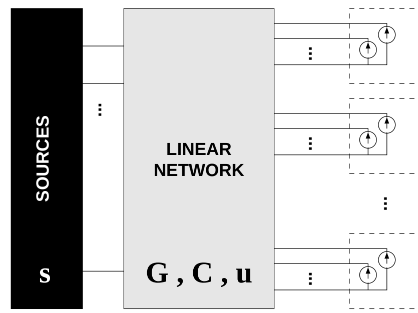

Figure 1: Network with nonlinear elements.

The general state variable reduction error function formulation presented here is a generalization of the piecewise harmonic balance technique [3, 1]. This formulation allows a flexible and convenient software implementation in a circuit simulator. Several circuit transient analysis methods are derived from this formulation; based on convolution, wavelets and time-marching schemes. The combination of the state variable reduction developed here with time marching schemes achieves an order of magnitude improvement in simulation speed compared with traditional circuit simulation methods [9].

We present the generalized circuit formulation in Section 2. Then we show how different circuit transient analysis techniques are derived from that formulation; wavelet-based in Section 3, convolution-based in Section 4 and time-marching-based in Section 5. The conclusions are given in Section 6.

2 Generalized Circuit Formulation

The formulation of the system equations begins with the partitioned network of Fig. 1 with the nonlinear elements replaced by variable voltage or current sources [3]. For each nonlinear element one terminal is taken as the reference and the element is replaced by a set of sources connected to the reference terminal. Both voltage and current sources are valid replacements for the nonlinear elements, but current sources are more convenient because they yield a smaller *modified nodal admittance matrix* (MNAM).

2.1 Linear Network

The MNAM of the linear subcircuit is formulated as follows. Define two matrices $\mathbf{G}$ and $\mathbf{C}$ of equal size n_m, where n_m is equal to the number of non-reference nodes in the circuit plus the number of additional required variables [4]. Define a vector $\mathbf{s}$ of size n_m for the right hand side of the system. The contributions of the independent sources and the nonlinear elements (which depend on the time t) will be entered in this vector. All conductors and frequency-independent MNAM stamps arising in the formulation will be entered in $\mathbf{G}$, whereas capacitor and inductor values and other values that are associated with dynamic elements will be stored in matrix $\mathbf{C}$. The linear system obtained is the following.

$$\mathbf{G}\mathbf{u}(t) + \mathbf{C}\frac{d\mathbf{u}(t)}{dt} = \mathbf{s}(t), \tag{1}$$

where $\mathbf{u}$ is the vector of the nodal voltages and selected currents. The source vector is

$$\mathbf{s}(t) = \mathbf{s}_f(t) + \mathbf{s}_v(t). \tag{2}$$

The $\mathbf{s}_f$ vector is due to the independent sources in the circuit. The $\mathbf{s}_v$ vector is the contribution of the currents of the nonlinear network.

2.2 Nonlinear Network

Let the nonlinear subnetwork be described by the following generalized parametric equations [1]:

$$\mathbf{v}_{NL}(t) = \mathbf{v}\left(\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \ldots, \frac{d^m\mathbf{x}}{dt^m}, \mathbf{x}_D(t)\right) \tag{3}$$

$$\mathbf{i}_{NL}(t) = \mathbf{i}\left(\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \ldots, \frac{d^m\mathbf{x}}{dt^m}, \mathbf{x}_D(t)\right) \tag{4}$$

where $\mathbf{v_{NL}(t)}$, $\mathbf{i_{NL}(t)}$ are vectors of voltages and currents at the ports of the nonlinear network, $\mathbf{x}(t)$ is a vector of state variables and $\mathbf{x}_D(t)$ a vector of time-delayed state variables, i.e., $[x_D(t)]_i = x_i(t - \tau_i)$. All vectors in Equations (3) and (4) have the same size equal to the number of ports of the nonlinear network. This kind of representation is convenient from the physical viewpoint, as it is equivalent to a set of implicit integro-differential equations in the port currents and voltages. This results in complete generality in device modeling. For example, it is no longer necessary to express nonlinear elements as voltage controlled current sources.

The error function of an arbitrary circuit is developed using connectivity information described by an incidence matrix and constitutive relations describing the nonlinear elements. The incidence matrix $\mathbf{T}$ relates the vectors of the linear and the nonlinear network equations

$$\mathbf{v}_{NL}(t) = \mathbf{T}\mathbf{u}(t) \tag{5}$$

$$\mathbf{s}_v(t) = \mathbf{T}^T\mathbf{i}_{NL}(t). \tag{6}$$

The matrix $\mathbf{T}$ is built as follows. The number of columns is n_m, and the number of rows is equal to the number of state variables, n_s. In each row, enter "+1" in the column corresponding to the positive terminal of the row nonlinear element port and "−1" in the column corresponding to the negative terminal (the local reference of the port). Then, each row of $\mathbf{T}$ has at most 2 nonzero elements and then the number of nonzero elements is at most $2n_s$.

2.3 Error Function Formulation

Combining Equations (1), (2), (6), the general equation for the linear network is obtained

$$\mathbf{G}\mathbf{u}(t) + \mathbf{C}\frac{d\mathbf{u}(t)}{dt} = \mathbf{s}_f(t) + \mathbf{T}^T\mathbf{i}_{NL}(t) \tag{7}$$

The reduced error function $\mathbf{f}(t)$ is defined as follows

$$\mathbf{f}(t) = \mathbf{T}\mathbf{u}(t) - \mathbf{v}_{NL}(t) = 0 \tag{8}$$

Equations (3), (4), (7) and (8) conform the generalized state variable reduction formulation. The error function in Equation (8) depends on the state variables and time,

$$\mathbf{f}\left(\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \ldots, \frac{d^n\mathbf{x}}{dt^n}, \mathbf{x}_D(t), t\right) = 0 \tag{9}$$

The dimension of the error function and the number of unknowns are equal to n_s, and this number is the minimum necessary to solve the equations of a circuit without any loss of information. This formulation is general and can be applied to derive several types of analysis. The discretization method used to approximate $\mathbf{x}(t)$ and $\mathbf{u}(t)$ and their derivatives determines the type of analysis, as it is shown in the following sections.

3 Transient Based on Wavelets

In the wavelet collocation method [5] the unknown function x is expanded in a wavelet series which must fit the circuit response at a number of collocation points. The equations combining the generalized state variable circuit formulation of Eq. (9) with the wavelet collocation method of Reference [5] were presented by the authors in References [6, 7].

Wavelets are introduced by considering the function $g(t)$ defined in an interval. The following square matrices $\mathbf{W}_J$ and $\mathbf{W}'_J$ can be defined:

$$\mathbf{g} = \mathbf{W}_J \hat{\mathbf{g}}_J, \quad \mathbf{g}' = \mathbf{W}'_J \hat{\mathbf{g}}_J \tag{10}$$

where $\mathbf{g}$, $\mathbf{g}'$ are vectors whose elements are the function and derivatives values, respectively, at the collocation points and $\hat{\mathbf{g}}_J$ is the vector of the corresponding coefficients. J is the maximum wavelet subspace level being considered. Define $\mathbf{M}_J$ as

$$\mathbf{M}_J = (\mathbf{G} \otimes \mathbf{W}_J + \mathbf{C} \otimes \mathbf{W}'_J),$$

where $\otimes$ is the Kronecker product (see Reference [6] for more details). The source vector $\mathbf{s}_{f,J}$ is obtained by expanding each element of $\mathbf{s}_f$ into the set of time samples corresponding to the collocation points. The first time sample of the source vector is replaced by the corresponding initial value. The linear circuit equation is then

$$\mathbf{M}_J \hat{\mathbf{u}}_J = \mathbf{s}_{f,J} + (\mathbf{T} \otimes \mathbf{I}_J) \mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) \tag{11}$$

where $\hat{\mathbf{u}}_J$ is the vector of the wavelet coefficients of the unknown circuit variables and $\mathbf{I}_J$ is an identity matrix of the same dimension of $\mathbf{W}_J$.

Let $\mathbf{x}_J$ be the state variable vector at all collocation points and $\hat{\mathbf{x}}_J$ the corresponding vector of coefficients in the transform domain. The first transform coefficient is excluded from the unknowns since it can be derived from the initial condition. Then we denote by $\mathbf{v}_{NL,J}(\hat{\mathbf{x}})_J$ and $\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J)$ the vectors of voltages and currents at the ports of the nonlinear devices at all the collocation points but the first. The error function $\mathbf{F}(\hat{\mathbf{x}}_J)$ is then, expanding (8)

$$\mathbf{F}(\hat{\mathbf{x}}_J) = (\mathbf{T} \otimes \mathbf{W}_J)\hat{\mathbf{u}}_J - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) = 0,$$

which can be expressed as

$$\mathbf{F}(\hat{\mathbf{x}}_J) = \mathbf{s}_{sv,J} + \mathbf{M}_{sv,J} \mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) = 0. \tag{12}$$

The $\mathbf{s}_{sv,J}$ vector and the $\mathbf{M}_{sv,J}$ matrix are defined as

$$\begin{aligned} \mathbf{s}_{sv,J} &= (\mathbf{T} \otimes \mathbf{W}_J)\mathbf{M}_J^{-1}\mathbf{s}_{f,J} \\ \mathbf{M}_{sv,J} &= (\mathbf{T} \otimes \mathbf{W}_J)\mathbf{M}_J^{-1}(\mathbf{T} \otimes \mathbf{I}_J). \end{aligned}$$

This wavelet transient formulation (Equation (12)) could also be modified to produce a formulation to find the periodic steady-state of a circuit. The only modification is that the equations relating the initial conditions are replaced by boundary condition equations.

4 Transient Based in Convolution

The state variable-based convolution transient analysis presented in [8] is derived from the following harmonic balance equations. If Equations (7) and (8) are expressed in the frequency domain, they become

$$\begin{aligned} &\mathbf{G}\mathbf{U}(f) + \mathbf{C}\boldsymbol{\Omega}(f)\mathbf{U}(f) = \\ &\qquad \mathbf{S}_f(f) + \mathbf{T}^T \mathbf{I}_{NL}(\mathbf{X}(f)) \\ &\mathbf{F}(\mathbf{X}(f), f) = \\ &\qquad \mathbf{T}\mathbf{U}(\mathbf{X}(f), f) - \mathbf{V}_{NL}(\mathbf{X}(f)) = 0 \end{aligned}$$

where f is the frequency and $\boldsymbol{\Omega}$ is the frequency domain derivation operator. Uppercase letters have been used to represent quantities in the frequency domain.

$$\boldsymbol{\Omega} = j\,2\pi f\,\mathbf{I},$$

Here j is the imaginary unit and $\mathbf{I}$ is the identity matrix. It can be shown that

$$\begin{aligned} &\mathbf{F}(\mathbf{X}(f), f) = \\ &\qquad \mathbf{S}_{SV}(f) + \mathbf{M}_{SV}(f)\mathbf{i}_{NL}(\mathbf{X}(f)) \\ &\qquad -\mathbf{V}_{NL}(\mathbf{X}(f), f) = 0, \end{aligned}$$

where

$$\mathbf{S}_{SV}(f) = \mathbf{T}[\mathbf{G} + \mathbf{C}\boldsymbol{\Omega}(f)]^{-1}\mathbf{S}_f(f) \tag{13}$$

and

$$\mathbf{M}_{SV}(f) = \mathbf{T}[\mathbf{G} + \mathbf{C}\boldsymbol{\Omega}(f)]^{-1}\mathbf{T}^T \tag{14}$$

In order to formulate the convolution transient analysis, the sources are included with the nonlinear subnetwork as they must be treated in the time domain. This implies $\mathbf{S}_{SV} = 0$. The number of state variables, n_s, is equal to the number of interface ports. The frequency domain voltages at the interface due to the linear network is

$$\mathbf{V}_L(\mathbf{X}, f) = \mathbf{M}_{SV}\mathbf{I}_{NL}(\mathbf{X}, f) \tag{15}$$

After discrete Fourier transformation, the i th component of $\mathbf{V}_L(f)$ at the discretized time step n_t, $(\mathbf{v}_L(n_t))_i$ is

$$(\mathbf{v}_L(n_t))_i = \sum_{j=1}^{n_s} (\mathbf{v}_L(n_t))_{i,j}$$

where $(\mathbf{v}_L(n_t))_{i,j}$ is defined as

$$(\mathbf{v}_L(n_t))_{i,j} = \mathcal{F}^{-1}\left[(\mathbf{M}_{SV})_{i,j}(\mathbf{I}_{NL})_j\right].$$

Here $\mathcal{F}^{-1}$ denotes the inverse discrete Fourier transformation. For the purposes of this transient analysis, $(\mathbf{v}_L(n_t))_{i,j}$ must be evaluated in the discrete time domain. The multiplication in the frequency domain becomes a convolution operation in the time domain.

The error function vector $\mathbf{f} = [f_1 \ldots f_i \ldots f_{n_s}]^{\mathrm{T}}$ can be formed at each time step, where

$$(f(\mathbf{x}, n_t))_i = \sum_{j=1}^{n_s} (\mathbf{v}_L(\mathbf{x}, n_t))_{i,j} - (\mathbf{v}_{NL}(\mathbf{x}, n_t))_i = 0 \quad (16)$$

The transient analysis proceeds as follows. Equation (16) is solved for $\mathbf{x}$ at each time step n_t. The value of $\mathbf{v}_{NL}(\mathbf{x}, n_t)$ is obtained directly from the nonlinear device models.

5 Time-Marching Transient

The system of differential equations (9) is converted into an algebraic system of nonlinear equations using time marching methods such as backward Euler or Trapezoidal integration. The following error function is obtained [9]

$$\mathbf{f}(\mathbf{x}_n) = \mathbf{s}_{sv,n} + \mathbf{M}_{sv}\mathbf{i}_{NL}(\mathbf{x}_n) - \mathbf{v}_{NL}(\mathbf{x}_n) = 0, \quad (17)$$

where $\mathbf{s}_{sv,n}$ is a vector and $\mathbf{M}_{sv}$ is a constant matrix. The size of the resulting algebraic system of nonlinear equations is $n_s \times n_s$. $\mathbf{M}_{sv}$ is constant if the time step is constant. $\mathbf{s}_{sv,n}$ changes at each time step. This is in contrast with traditional circuit simulators, where the number of simultaneous unknowns is equal to the number of non-reference nodes in the circuit plus the number of additional required variables (n_m). Therefore this formulation is convenient for microwave circuits where normally $n_s \ll n_m$. A transient analysis implementation based on this formulation achieves more than an order of magnitude improvement for some simulation times compared with traditional circuit simulation methods [9].

6 Conclusions

The circuit formulation described in this paper constitutes a reduction method because the size of the nonlinear problem is reduced from the number of nodal unknowns, n_m, to the number of state variables, n_s. In contrast, traditional methods must solve for n_m unknowns which is typically much greater than n_s in microwave circuits. Thus the proposed method is often more convenient. The state-variable approach provides great flexibility for the design of nonlinear device models. The state variables can be chosen to achieve robust numerical characteristics. We have shown that the generalized formulation is a theoretical tool that can be used to derive a set of equations for different types of analysis.

References

[1] V. Rizzoli, A. Lipparini, A. Costanzo, F. Mastri, C. Ceccetti, A. Neri and D. Masotti, “State-of-the-art harmonic-balance simulation of forced nonlinear microwave circuits by the piecewise technique,” *IEEE Trans. on Microwave Theory and Tech.*, Vol. 40, No. 1, Jan 1992, pp. 12–28.

[2] C. E. Christoffersen, U. A. Mughal and M. B. Steer, “Object oriented microwave circuit simulation,” *Int. J. of RF and Microwave Computer-Aided Engineering*, Vol. 10, Issue 3, 2000, pp. 164–182.

[3] C. E. Christoffersen, M. B. Steer and M. A. Summers, “Harmonic balance analysis for systems with circuit-field interactions,” *1998 IEEE Int. Microwave Symp. Dig.*, June 1998, pp. 1131–1134.

[4] J. Vlach and K. Singhal, *Computer Methods for Circuit Analysis and Design*, Van Nostrand Reinhold, 1994.

[5] D. Zhou and W. Cai, “A fast wavelet collocation method for high-speed VLSI circuit simulation,” *IEEE Trans. on Circuits and Systems—I: Fundamental Theory and Appl.*, Vol. 46, Aug. 1999, pp. 920–930.

[6] C. E. Christoffersen and M. B. Steer, “State-variable-based transient circuit simulation using wavelets,” *IEEE Microwave and Guided Wave Letters*, Vol. 11, April 2001, pp. 161–163.

[7] C. E. Christoffersen and M. B. Steer, “Comparison of wavelet- and time-marching-based microwave circuit transient analyses,” *2001 IEEE Int. Microwave Symp. Dig.*, May 2001, pp. 447–450.

[8] C. E. Christoffersen, M. Ozkar, M. B. Steer, M. G. Case and M. Rodwell, “State variable-based transient analysis using convolution,” *IEEE Transactions on Microwave Theory and Tech.*, Vol. 47, June 1999, pp. 882–889.

[9] C. E. Christoffersen *Global modeling of nonlinear microwave circuits*, Ph. D. Dissertation, North Carolina State University, December 2000.