0.0.1 Mutual Inductor **K**

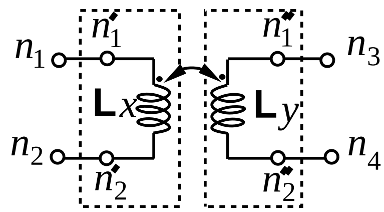

Figure 1: K — Mutual inductor element.

`SPICE` *Form:*

`K`*name* `L`*name1* `L`*name2* *CouplingValue*

`L`*name1*	is the name of the first inductor of the coupled inductor list. The first node of `L`*name1* is dotted using the dot convention. In the mutual coupled inductor model (the default model) the value of `L`*name1* is the self inductance L_1. In the transformer `CORE` model (which is used if a *ModelName* is supplied) the value of `L`*name1* is the number of turns N_1. (Note, *ModelName* cannot be specified with the SPICE2G6 and SPICE3 simulators.)(Required)
`L`*name2*	is the name of the second inductor in the coupled inductor list. The first node of `L`*name2* is dotted using the dot convention. In the mutual coupled inductor model the value of `L`*name2* is the self inductance L_2. In the transformer `CORE` model (which is used if a *ModelName* is supplied the value of `L`*name2* is the number of turns N_2. (SPICE3: Required.)
`L`*nameN*	is the name of Nth inductor in the coupled inductor list. The first node of `L`*nameN* is dotted using the dot convention. In the mutual inductor model the value of `L`*nameN* is the self inductance L_N. In the transformer `CORE` model (which is used if a *ModelName* is supplied the value of `L`*name2* is the number of turns N_N. Not valid in SPICE2G6 or SPICE3 for $N > 2$.
CouplingValue	is the coefficient of mutual coupling of the inductors. (Units: none; Required; Symbol: K_{COUPLING}; $0 < K_{\text{COUPLING}} \leq 1$)
ModelName	is the optional model name. PSPICE only.
Size	is the size scaling factor. It scales the magnetic cross-section and represents the number of lamination layers. (Units: none; Optional; Default: 1; Symbol: *Size*)

Example:

```
K43 LAA LBB 0.999
KXFRMR L1 L2 0.87
```

Description:

Model Type

IND PSpice only

The mutual coupled inductor model represents coupled inductors by self inductances L_i and mutual inductances M_{ij}. This is the model used in SPICE2G6 and spicthree and in PSPICE if a CORE model is not supplied. Here L_i is the self inductance of the ith inductor element and M_{ij} is the mutual inductance of the ith and jth inductor elements. The mathematical model of the coupled element consists of voltage sources controlled by the time derivatives of current. If two inductors are coupled

$$V_1 = L_1 \frac{dI_1}{dt} + M_{12} + \frac{dI_2}{dt} \tag{1}$$

and

$$V_2 = L_2 \frac{dI_2}{dt} + M_{21} + \frac{dI_1}{dt} \tag{2}$$

If N inductors are coupled, as supported in PSPICE , the mathematical model is

$$V_i = L_i \frac{dI_i}{dt} + \sum_{\substack{j=1 \\ j \neq i}}^{N} M_{ij} \frac{dI_j}{dt} \tag{3}$$

The mutual inductance M_{ij} is determined from the self-inductances L_i and L_j of the inductors and the coupling coefficient K_{COUPLING} supplied as an element parameter by

$$K_{\mathrm{COUPLING}} = \sqrt{\frac{M_{ij}}{L_i L_j}} \tag{4}$$

K_{COUPLING} may have any value between 0 and 1 including 1. Ferrite core provides almost ideal coupling with $K = 0.999$ or higher.

In SPICE2G6 and SPICE3 a transformer with several coils must be represented by several K elements. For example, a transformer with one primary and two secondaries is specified as

```
* PRIMARY
L1 1 2 100U
* FIRST SECONDARY
L2 3 4 100U
* SECOND SECONDARY
L3 5 6 100U
* TRANSFORMER
K1 L1 L2 0.999
K2 L1 L3 0.999
K2 L2 L3 0.999
```

In PSPICE the transformer above can be either represented using the SPICE2G6 and SPICE3 format above or by the more compact format

```
* PRIMARY
L1 1 2 100U
* FIRST SECONDARY
L2 3 4 100U
* SECOND SECONDARY
L3 5 6 100U
* TRANSFORMER
K1 L1 L2 L3 0.999
```

CORE Model PSPICE Only Magnetic Core Model

CORE

Form

`.MODEL` *ModelName* `CORE(` [[*keyword* = *value*] ...] `)`

Example

```
.MODEL TRANSFORMER CORE(AREA=1 PATH=9.8 GAP=0.1 MS=1.250M)
```

Model Keywords

Name	Description	Units	Default
A	shape parameter (A)	A/M	10^3
ALPHA	(α) interdomain coupling parameter	-	0.001
AREA	mean magnetic crossection ($Area$)	cm^2	0.1
GAMMA	domain damping parameter. (γ)	-	∞
C	domain flexing parameter (C)	-	0.2
GAP	effective air-gap length (L_{GAP})	cm	0
K	domain anisotopy parameter (pinning constant) (K)	A/M	500
MS	magnetization saturation (M_S)	A/M	10^6
PACK	pack (stacking) factor (F_{PACK})	cm	0
PATH	mean magnetic path length in the core (L_{PATH})	cm	1

The `CORE` model models a transformer core. It is assumed that the model parameters were determined or measured at the nominal temperature T_{NOM} (default $27°C$) specified in the most recent `.OPTIONS` statement preceeding the `.MODEL` statement.

The `CORE` model uses the Jiles-Atherton model described in [**?**]. This model is based on domain wall motion and includes flexing of the domain wall, interdomain coupling, coercivity, remanence and magnetic saturation. Hysteresis due to domain wall pinning at defect sites is modeled. This impedance to domain wall motion dominates the characteristics of magnetic devices.

As with the default mutually coupled inductor model, the `CORE` model calculates the voltage across the ith set of windings from the total ampere turns which is the magnetomotive force MMF. Thus

$$V_i = \frac{d\phi_i}{dt} = f(MMF) \tag{5}$$

where

$$MMF = \sum_{j=1}^{N} N_j I_j \tag{6}$$

Here the number of turns of the jth winding, N_j, is the "*InductanceValue*" of L_j the name of which is the jth `L`*name* given on the `K` element line. I_i is the curent flowing through the ith winding. A_{TURNS} produces the magnetic field H_{CORE} in the core. This in turn produces the B field. The B field is proportional to the flux, in the core and hence to the voltage V_i. The relationship between B and H in the core is nonlinear and hysteretic. The airgap also affects the B-H relationship.

Air-Gap Effect

Along the complete magnetic path

$$H_{\mathrm{CORE}} L_{\mathrm{PATH}} + H_{\mathrm{GAP}} L_{\mathrm{GAP}} = MMF \tag{7}$$

where H_{CORE} is the magnetic field in the core and H_{GAP} is the magnetic field in the air gap. L_{PATH} and L_{GAP} are the model parameters `PATH` and `GAP`. If the air gap is small then all of the flux in the core passes through the air gap so that $B_{\mathrm{GAP}} = B_{\mathrm{CORE}}$. In the air-gap the magnetization is negligible so that $B_{\mathrm{GAP}} = H_{\mathrm{GAP}}$

This leads to a relationship between the B and H fields in the core:

$$H_{\mathrm{CORE}} L_{\mathrm{PATH}} + B_{\mathrm{CORE}} L_{\mathrm{GAP}} = MMF \tag{8}$$

It is a simple matter to solve for B_{CORE} and H_{CORE} if L_{GAP} =0 as then

$$H_{\mathrm{CORE}} = \frac{MMF}{L_{\mathrm{PATH}}} \tag{9}$$

If $L_{\mathrm{GAP}} > 0$ then (8) must be solved in conjunction with the relationship between H_{CORE} and magnetization M in the core. This relationship is based on the theory of loosely coupled domains developed by Jiles and Atherton.

Jiles-Atherton Model

The B-H curve of a magnetic material biased by `AC` and `DC` magnetic fields is called the anhysteric and is mathematically described by the Jiles-Atherton model. This model determines an anhysteric magnetization M_{AN} which is related to the saturation magnetization M_S by

$$M_{\mathrm{AN}} = M_S \left[\coth\left(\frac{H_{\mathrm{EFF}}}{Size\,A} \right) - \frac{Size\,A}{H_{\mathrm{EFF}}} \right] \tag{10}$$

where A is the shape parameter and the effective field in the core

$$H_{\mathrm{EFF}} = H_{\mathrm{CORE}} + \alpha M_{\mathrm{AN}} \tag{11}$$

Here H is the magnetizing influence. Domain wall flux is magnetic current which is proportional to the change in magnetization. The change in magnetization consists of a reversible component due to flexing of the domain walls and an irreversible component due to movement of domain walls from one pinning location to another. Energy is dissipated (hence the motion is irreversible) in moving the domain wall from one pinning location to another but energy is stored (hence reversible) when the domain wall flexs. This is mathematicly modeled by

$$\frac{dM}{dH_{\mathrm{CORE}}} = \left(\frac{dM}{dH_{\mathrm{CORE}}} \right)_{\mathrm{REVERSIBLE}} + \left(\frac{dM}{dH_{\mathrm{CORE}}} \right)_{\mathrm{IRREVERSIBLE}} \tag{12}$$

where the reversible component

$$\left(\frac{dM}{dH_{\mathrm{CORE}}} \right)_{\mathrm{REVERSIBLE}} = C \frac{d(M_{\mathrm{AN}} - M)}{dH} \tag{13}$$

and the irreversible component

$$\left(\frac{dM}{dH_{\mathrm{CORE}}} \right)_{\mathrm{IRREVERSIBLE}} = \frac{M_{\mathrm{AN}} - M}{K} \tag{14}$$

where K is the pinning energy per volume and is akin to mechanical drag. M and H_{CORE} are found by solving (12) and (8) simultaneously.

The small signal relative permeability of the core is

$$\mu_r = \left\{ \left[\left(\frac{dM}{dH_{\mathrm{CORE}}} + 1 \right) F_{\mathrm{PACK}} \right]^{-1} + \frac{L_{\mathrm{GAP}}}{L_{\mathrm{PATH}}} \right\}^{-1} \tag{15}$$

and the flux passing through the ith winding is

$$\phi_i = \mu_0 (M + H_{\mathrm{CORE}}) N_i F_{\mathrm{PACK}}\, Size\, Area \tag{16}$$

The voltage across the ith winding is then found as

$$V_i = \frac{d\phi_i}{dt} \tag{17}$$

AC Analysis

For `AC` analysis the mutual inductor model is used even if a `CORE` model is specified. This allows a different coefficient of mutual coupling to be used in `AC` analysis than would otherwise be determined by nonlinear model evaluation.

Noise Analysis

The `K` element does not contribute to noise.

Notes:
There is no equivalent element in *f*REEDA™.

Credits:

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